A Projection Method Based Fast Transient Solver for Incompressible Turbulent Flows

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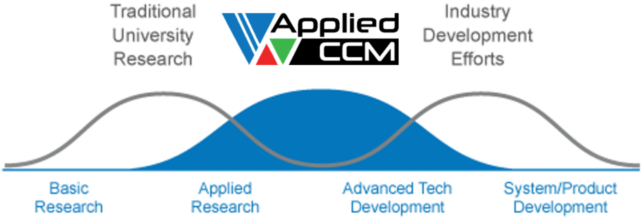
Applied CCM

- Specialize in the application, development and support of OpenFOAM® - based software
- Creators and maintainers of Caelus
- Locations: Canada, Australia, USA



Motivation

Why develop another transient solver?

- DES and LES attractive because RANS tends to be problem specific
- Low cost hardware + open-source software $\Rightarrow$ DES and LES feasible
- Traditional transient, incompressible algorithms (PISO and SIMPLE) do not scale well for large HPC, GPU and Many Integrated Core (MIC) environments
- Let's review PISO algorithm

PISO Overview

Pressure Implicit with Splitting of Operators (PISO)[1] method:

1. Solve momentum equation *(predictor step)*
2. Calculate intermediate velocity, u^* (pressure dissipation added)
3. Calculate momentum fluxes
4. Solve pressure equation: $\nabla \cdot (\frac{1}{A_p} \nabla p) = \nabla \cdot u^*$
5. Correct momentum fluxes
6. Correct velocity *(corrector step)*

Repeat steps 2 – 6 for PISO (1 – 6 for transient SIMPLE)

[1] Isaa, R.A. 1985, “Solution of the implicitly discretised fluid flow equations by operator splitting” *J. Comp. Phys.*, **61**, 40.

Fractional Step Error

- Step 2 main issue with PISO
- Predicted velocity used only to update matrix coefficients: $u^* = \frac{1}{a_p}\left(\Sigma\, a_{nb}\, u_{nb} - (\nabla p - \overline{\nabla p})\right)$
- Pseudo-velocity, u^*, is used on the RHS of pressure equation
- Therefore requires at least two corrections to make velocity and pressure consistent

Pressure Matrix

- Non-constant coefficients ($\frac{1}{a_p}$) in pressure matrix affects multi-grid solver performance
- Multi-grid agglomeration levels cached first time pressure matrix assembled
- Coefficients ($\frac{1}{a_p}$) only valid for the first time step
- Turning off caching of agglomeration too expensive

SLIM Overview

Semi Linear Implicit Method (SLIM), based on projection method[1]: decompose velocity into vortical and irrotational components.

1. Solve momentum equation (vortical velocity)
2. Calculate momentum fluxes (pressure dissipation added)
3. Solve pressure equation (irrotational velocity):
   $\Delta t \, \nabla^2(p) = \nabla \cdot u$
4. Correct momentum flux
5. Correct velocity (solenoidal)

Use incremental pressure approach to recover correct boundary pressure

[1] Chorin, A.J. 1968, "Numerical Solution of the Navier-Stokes Equations",*Mathematics of Computation* **22**: 745-762

Fractional Step Error

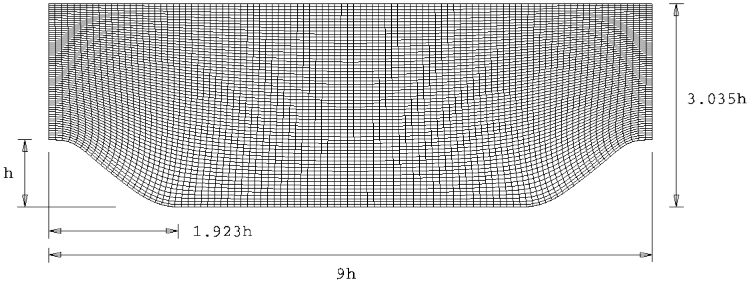
- Velocity split into vortical and potential components - much smaller fractional step error
- Pressure and velocity maintain stronger coupling
- Continuity satisfied within one pressure solve because predicted velocity used directly in pressure equation

Pressure Matrix

- Pressure matrix coefficients purely geometric
- Multi-grid agglomeration levels assembled during first step now consistent for all time steps
- Significantly improves parallel scalability for multi-grid solver

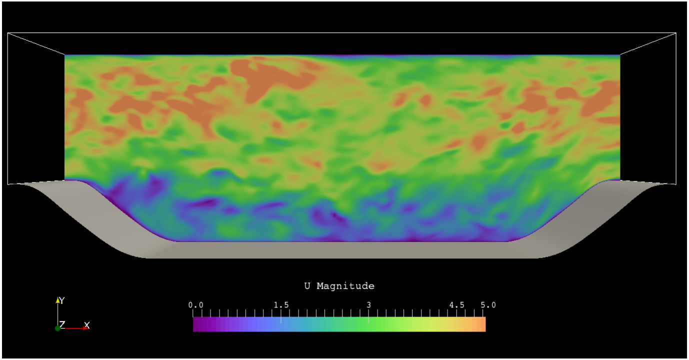
2D Periodic Hills

- Two dimensional, stream-wise, staggered hills of polynomial shape
- Re_h = 10,595
- Stream-wise and span-wise boundaries periodic. Hills and top boundaries no slip.
- Grid: $\sim$ 4.5 million hex cells; LES model: Smagorinsky



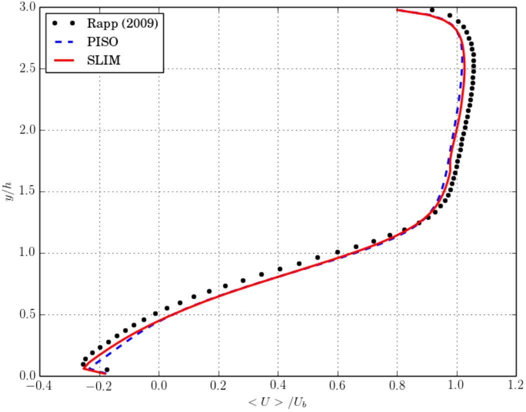
Validation

- Experimental data of Rapp (2009)
- Mean and second moment components at 10 vertical rakes



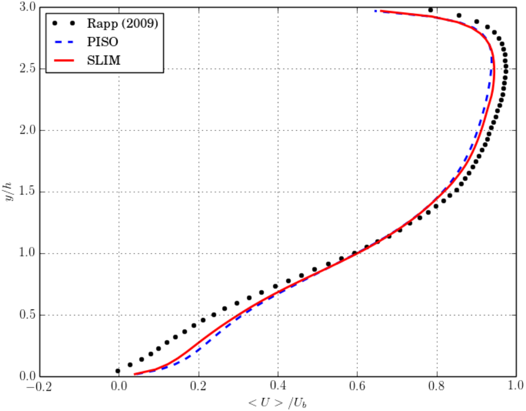
x/h = 2

- Both compare favorably
- SLIM slightly closer than PISO



x/h = 4

- SLIM consistently closer than PISO at all locations
- Likely due to lower fractional step error



Simulation Time

- SLIM on average about 30% faster on modest HPC system
- Fewer total iterations of pressure equation (SLIM: 10; PISO: 14)

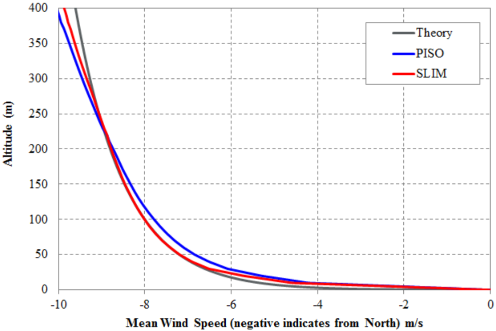
# cores	PISO	SLIM	% diff.
1	2095	1550	26
5	988	711	28
10	419	302	28
20	330	231	30
40	219	147	33
60	216	138	36

Precursor Simulation

- Establish turbulent conditions to use as initial condition for wind park simulation
- Start from quiescent condition. Run until fully turbulent.
- Steam-wise and span-wise periodic
- Grid size: 50 million hex cells
- Results courtesy of Greg Oxley at Vestas using Firestorm super computer

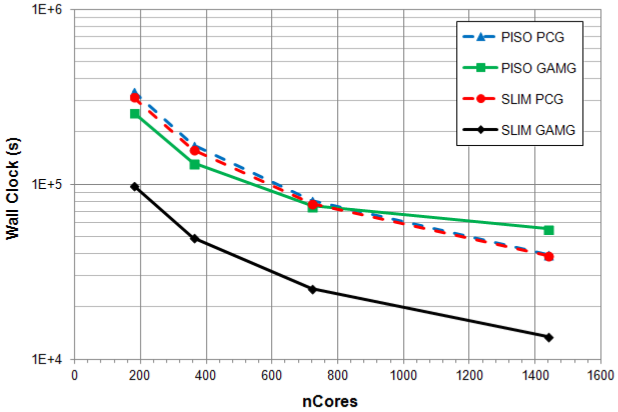
Mean Wind Profile

- SLIM slightly more accurate than PISO
- Fully turbulent condition reached sooner than PISO



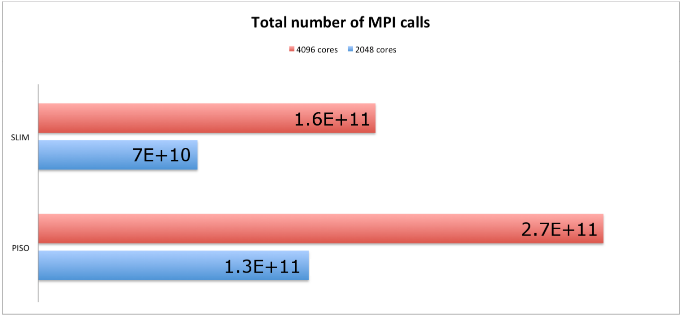
Scaling

- Consistent multi-grid agglomeration levels give SLIM significant advantage



MPI Profiling

- Profiled MPI calls on 125 million cell mesh up to 4096 cores



Future Work

- For static grids, pressure matrix construction may be pulled entirely from time loop to save assembly of pressure matrix every time step
- Advantageous for GPU and MIC computing. Compute pressure matrix once. Only need to transfer RHS vector
- For peta-scale core counts, solve momentum equations explicitly (Runga-Kutta). Combined with above, could perform close to fully explicit codes
- Solvers have been developed and are undergoing testing

Summary

- SLIM significantly faster than PISO. Problem dependent but 30-100% is typical improvement and even more for very large HPC calculations.
- Exact velocity splitting improves both convergence and accuracy
- Geometric pressure matrix coefficients advantageous for parallel efficiency, particularly for multi-grid solvers
- Additional modifications enable scaling to very large number of cores (HPC, GPU, MIC)

Strategic Perspective

Select research and development projects that are unique and help transfer knowledge to industrial applications.

- Solvers: transient, compressible, multi-phase, combustion, acoustics
- Turbulence: RANS, DES and LES, **VLES**, wall models
- Sensitivity, design optimisation, and uncertainty propagation: adjoint, **tangent**
- Numerical acceleration and stabilisation
- Platforms and architectures: HPC, GPU, MIC