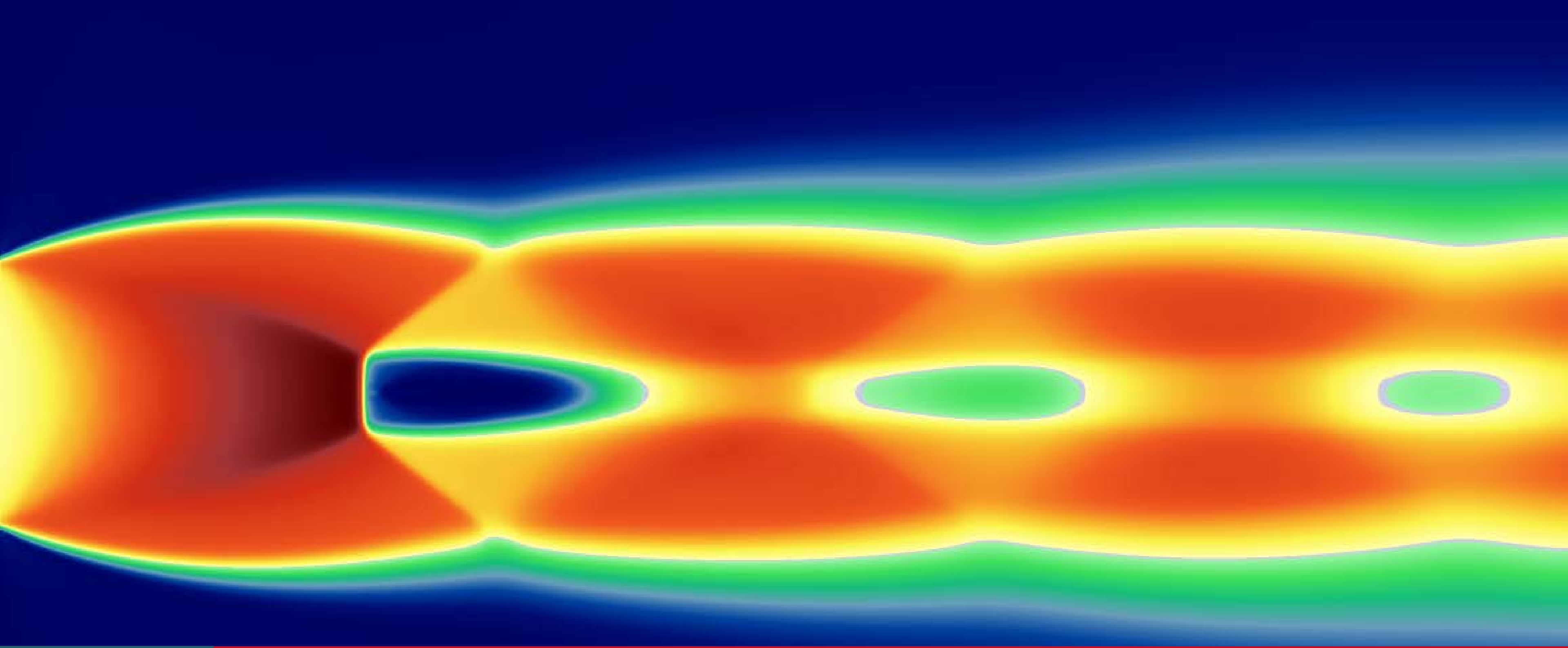

Velocity Correction Scheme for All Speed Flows

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Prof. Aleksandar Jemcov

Introduction

- Development of pressure based all Mach number compressible solver exhibited smearing of shocks.
- Rhie-Chow interpolation is culprit.
  - Nerinckx, K., Vierendeels, J. and Dick, E. (2004), “A Mach-uniform pressure correction algorithm using AUSM flux definitions”, Advances in Fluid Mechanics V.
  - Darwish, M. and Moukalled, F. (2014), “A Fully Coupled Navier-Stokes Solver for Fluid Flow at All speeds”, Numerical Heat Transfer, Part B.
- Develop a scheme closer to density based approach but in a pressure based solver.
- New solver does’t use Rhie-Chow interpolation,
  - Alternative dissipation scheme based on Kurganov-Tadmor scheme combined with the projection operator applied to continuity equation.
- Intent to keep formulation as close to rhoCentralFoam solver implementation of KT scheme.

Solver

- Pressure velocity coupling through a projection operator

$$\vec{u} = \widehat{u} - \frac{\delta t \nabla p}{\rho}$$

- KNP Variation of the KT scheme used (KTNP)

$$\Psi_{j+\frac{1}{2}}\left(\vec{u}_{j+\frac{1}{2}} \cdot \vec{S}_f\right) = \frac{a^{+}_{j+\frac{1}{2}} f\left(u^{-}_{j+\frac{1}{2}}\right) - a^{-}_{j+\frac{1}{2}} f\left(u^{+}_{j+\frac{1}{2}}\right)}{a^{+}_{j+\frac{1}{2}} - a^{-}_{j+\frac{1}{2}}} + \frac{a^{+}_{j+\frac{1}{2}} a^{-}_{j+\frac{1}{2}}}{a^{+}_{j+\frac{1}{2}} - a^{-}_{j+\frac{1}{2}}}\left[\Psi^{+}_{j+\frac{1}{2}} - \Psi^{-}_{j+\frac{1}{2}}\right]$$

Solver cont'd

- Solver algorithm resembles the projection algorithm widely used for incompressible flow.
  - Solve momentum predictor
  - Solve energy equation
  - Solve pressure equation

$$\frac{d(\psi p)}{dt}V + \sum_f \rho_f^P \alpha_f^P \vec{u}_f^{\,P} \cdot \vec{S}_f + \sum_f \rho_f^N \alpha_f^N \vec{u}_f^{\,N} \cdot \vec{S}_f - \sum_f (\psi p)_f^P \alpha_f^P a_f^{\min}$$

$$+\sum_f (\psi p)_f^N \alpha_f^N a_f^{\min} - \sum_f \alpha_f^P \delta t (\nabla p)_f^P \cdot \vec{S}_f - \sum_f \alpha_f^P \delta t (\nabla p)_f^P \cdot \vec{S}_f$$

$$= -\sum_f (\psi p)_f^P \alpha_f^P \left(\frac{\delta t \nabla \widehat{p}}{\rho}\right)_f^P \cdot \vec{S}_f - \sum_f (\psi p)_f^N \alpha_f^N \left(\frac{\delta t \nabla \widehat{p}}{\rho}\right)_f^N \cdot \vec{S}_f$$

  - Correct velocity

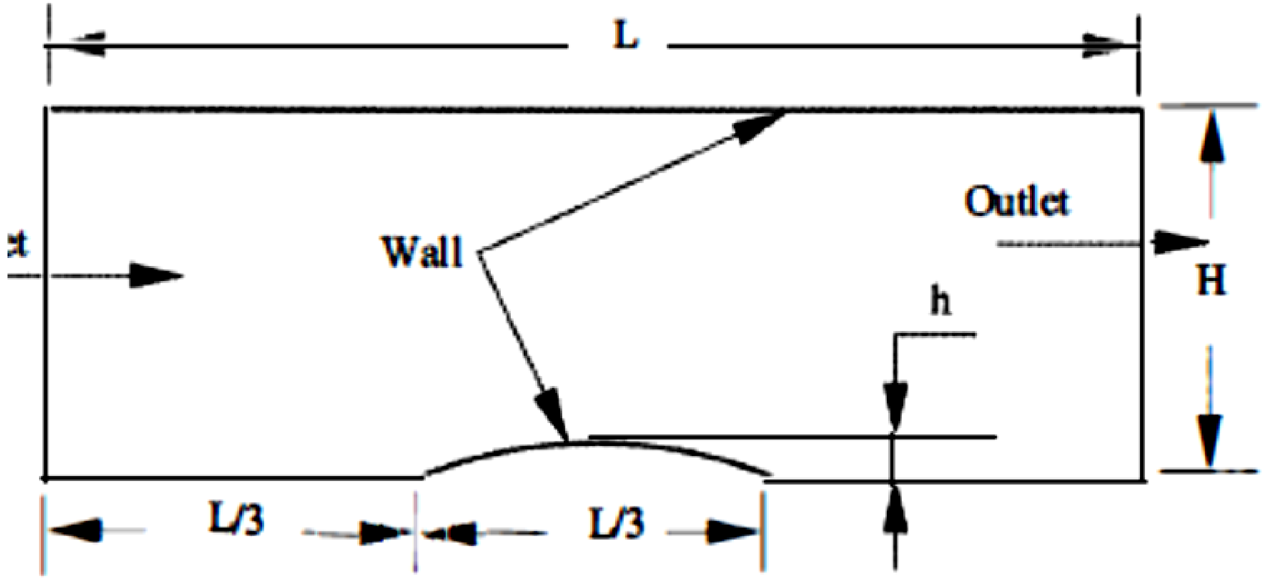
Solver cont'd

- KTNP scheme is not suitable for low speed flows.
- Change the formulation of the flux interpolation
  - Low speed – linear interpolation
  - High speed – KTNP scheme interpolation
  - Change is achieved using switching function depending on Mach number

- The switching function is defined as

$$\lambda\left(Ma\right)=\frac{1}{2}\left[1+\tanh\left(\frac{Ma-Ma_t}{Ma_\delta}\right)\right]$$

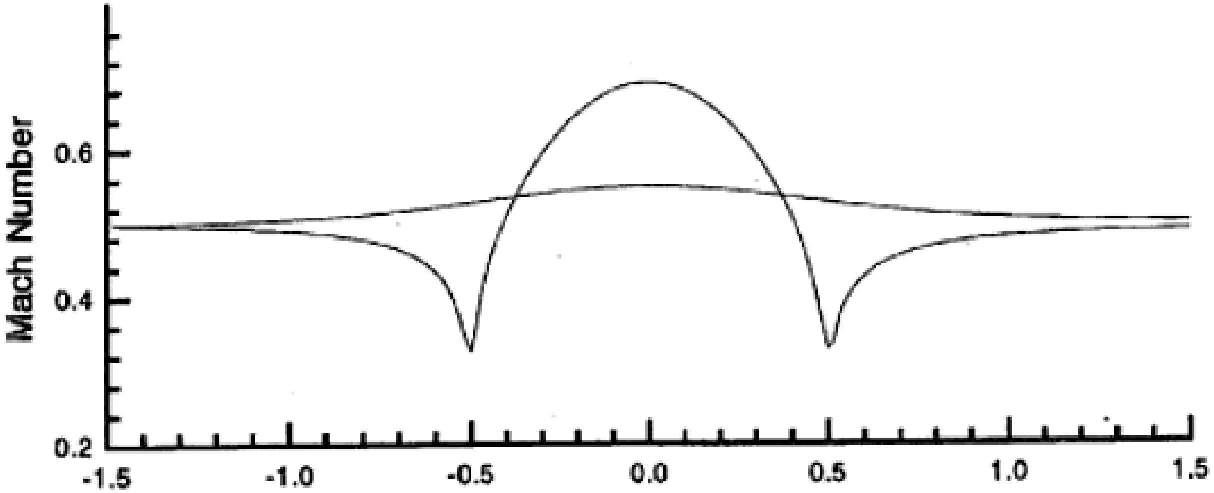
Steady state 2D bump

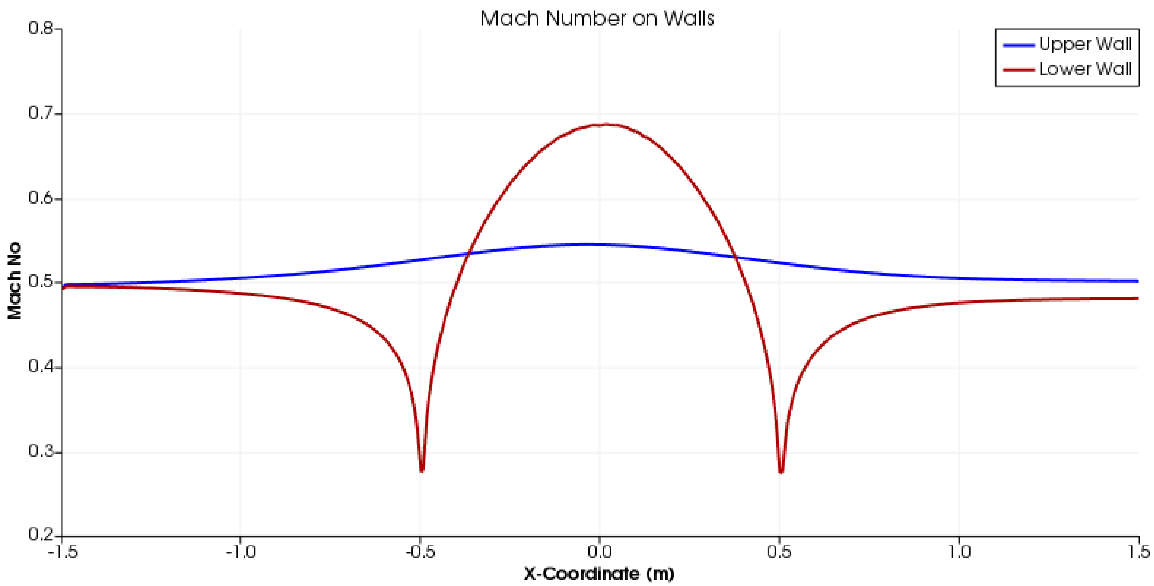


- Three Mach number cases
  - Subsonic - 0.5
  - Transonic - 0.675
  - Supersonic - 1.4
- Comparison with solutions from Riemann solvers
  - Favini, B. and Broglia, R. (1996) ”Multigrid acceleration of second-order ENO schemes from low subsonic to high supersonic flows”, Int. J. Num. Meth. in Fluids.

Subsonic inviscid flow over a 2D bump

(Ma = 0.5)





Subsonic inviscid flow over a 2D bump
Pressure

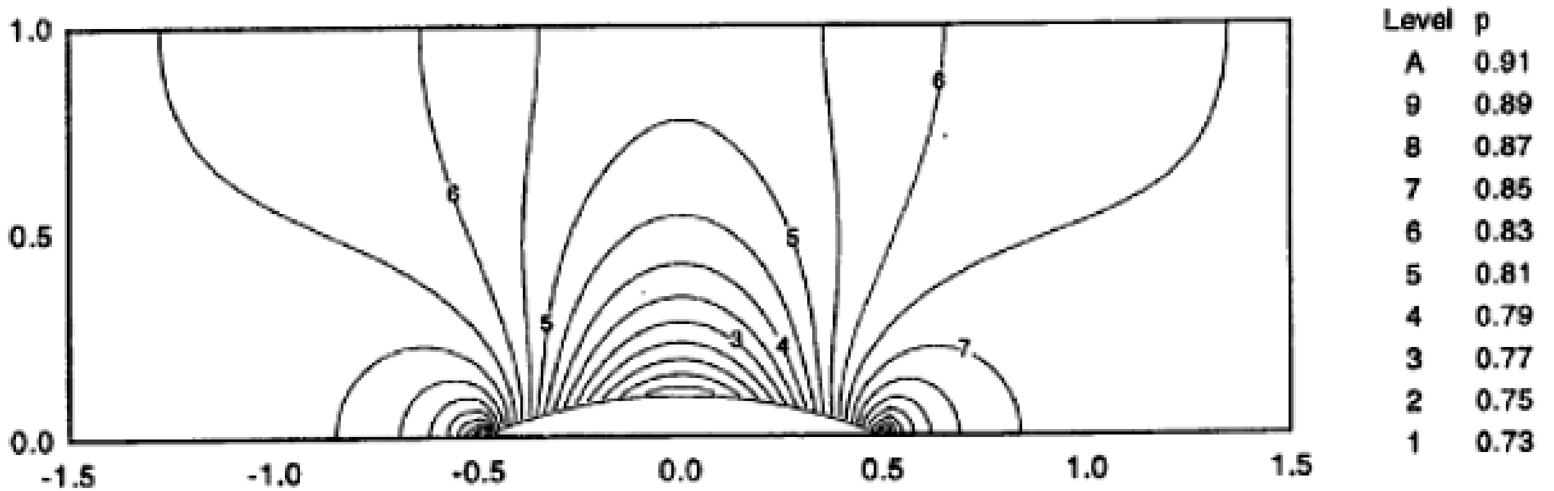
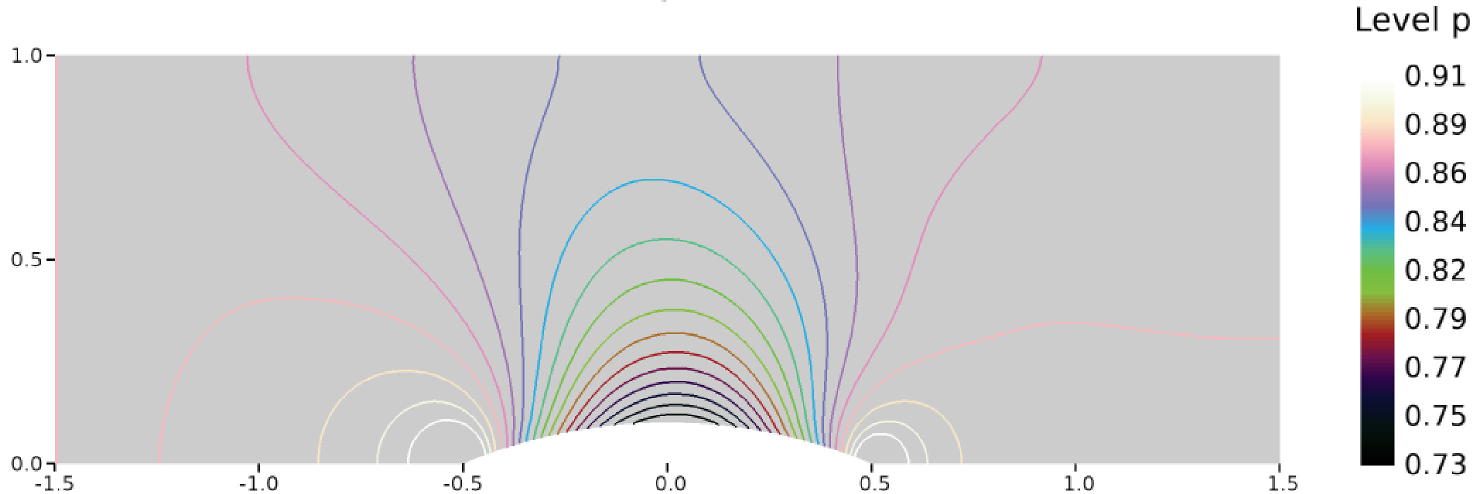
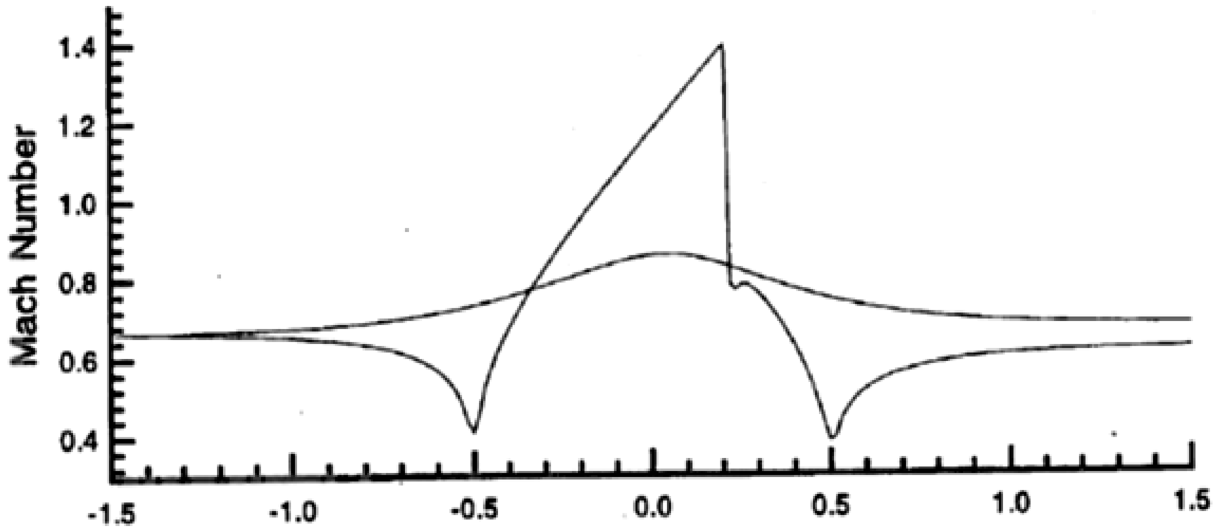


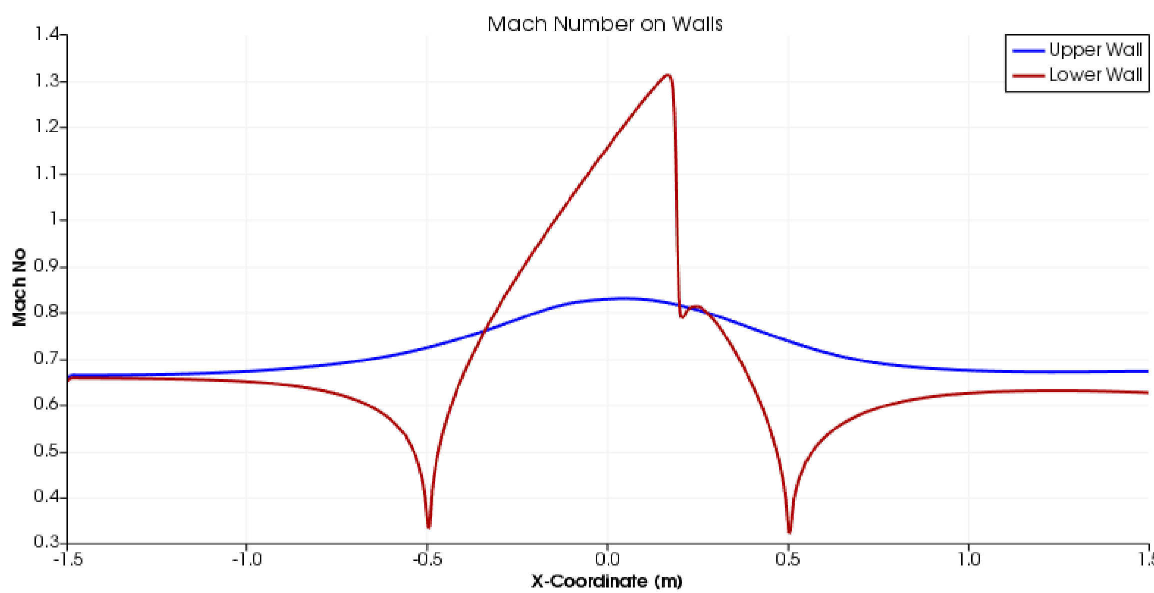
Figure 3. Subsonic flow in channel: Mach number distribution on upper and lower walls (top) and pressure contours (bottom). Exact solver, grid 192 × 64, $M_\infty = 0{\cdot}5$



Transonic inviscid flow over a 2D bump

(Ma = 0.675)





Transonic inviscid flow over a 2D bump
Pressure

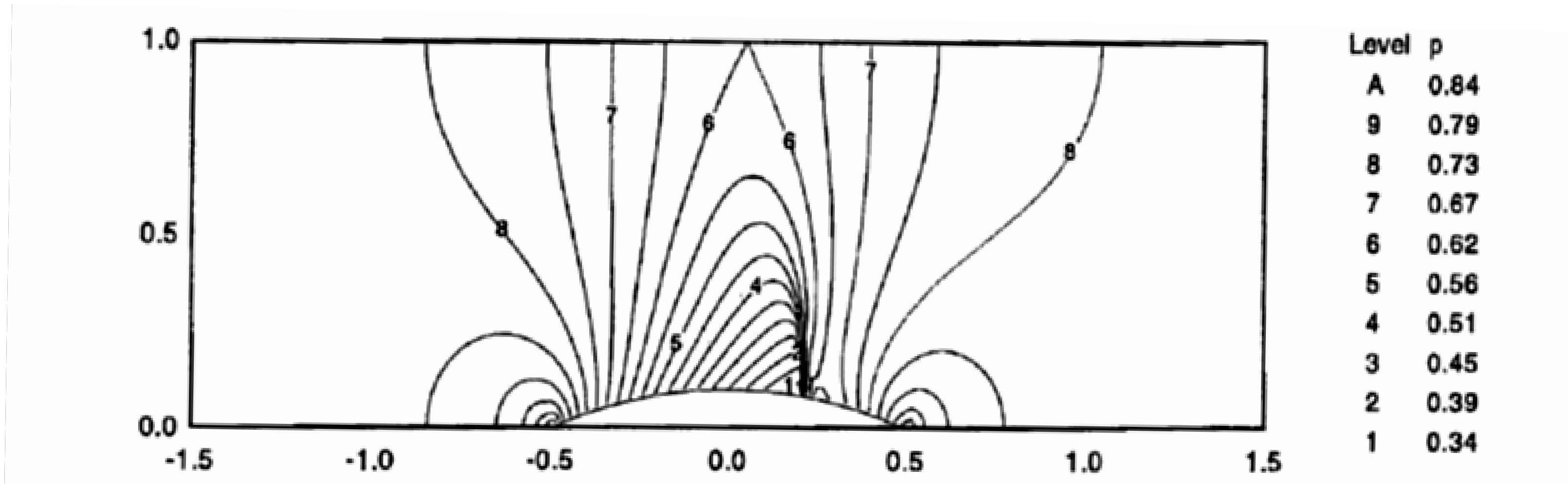
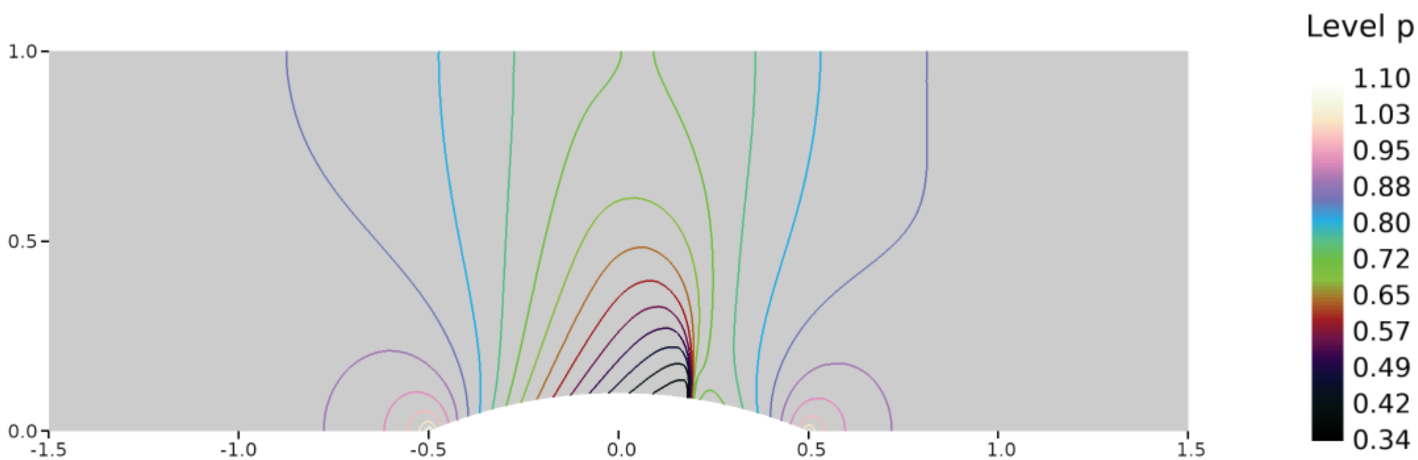
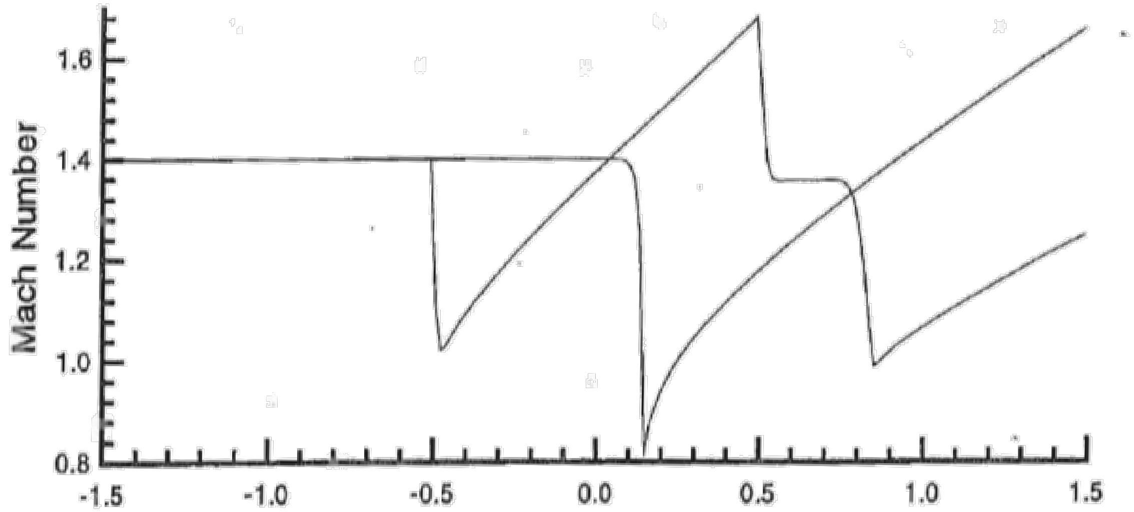


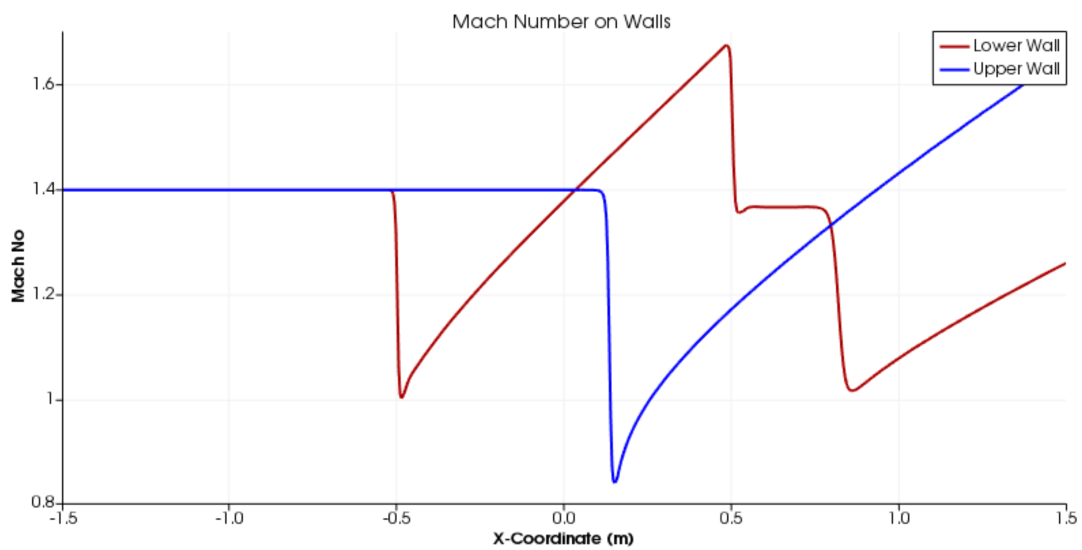
Figure 5. Transonic flow in channel: Mach number distribution on upper and lower walls (top) and pressure contours (bottom). Exact solver, grid 192 × 64, $M_\infty = 0{\cdot}675$



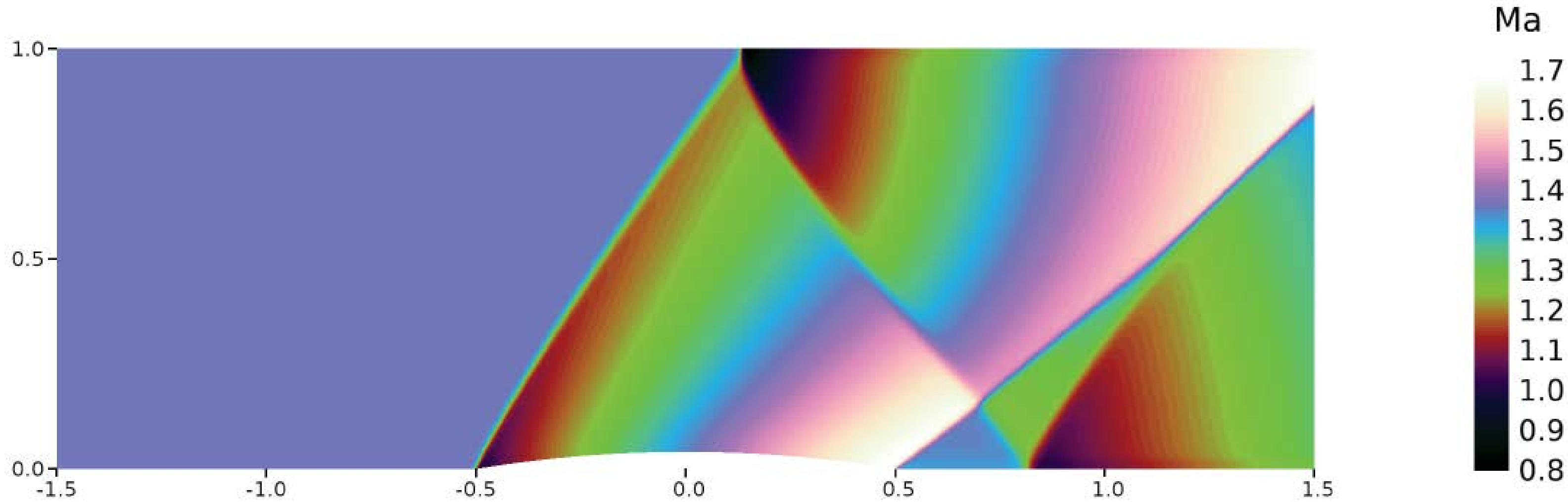
Supersonic inviscid flow over a 2D bump

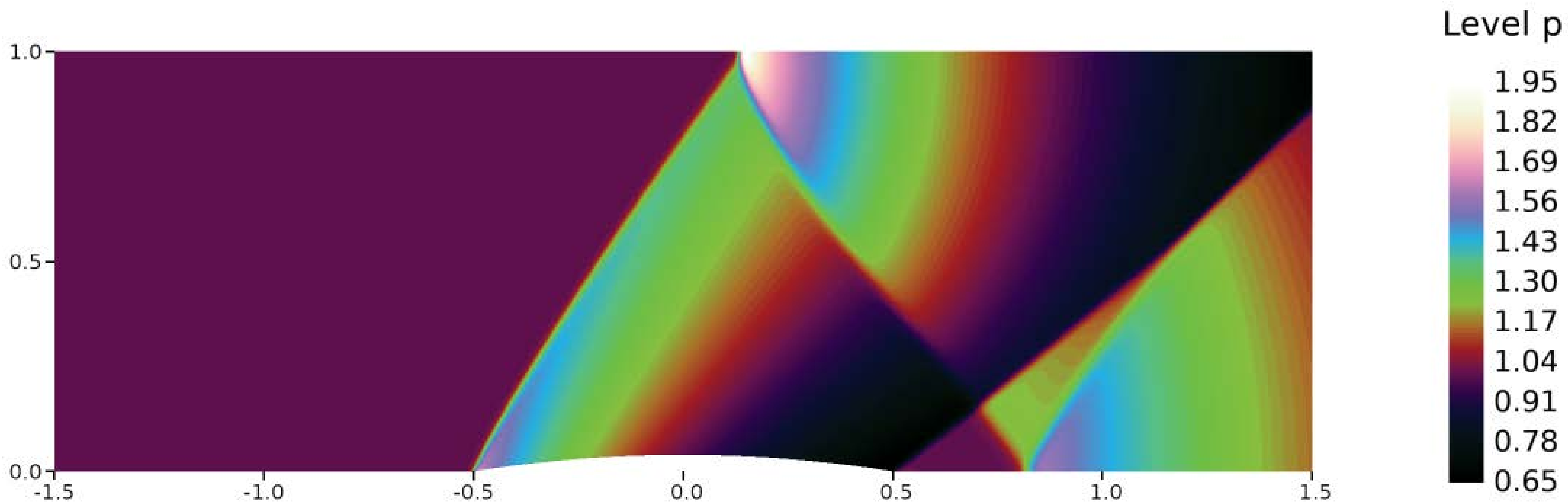
(Ma = 1.4)



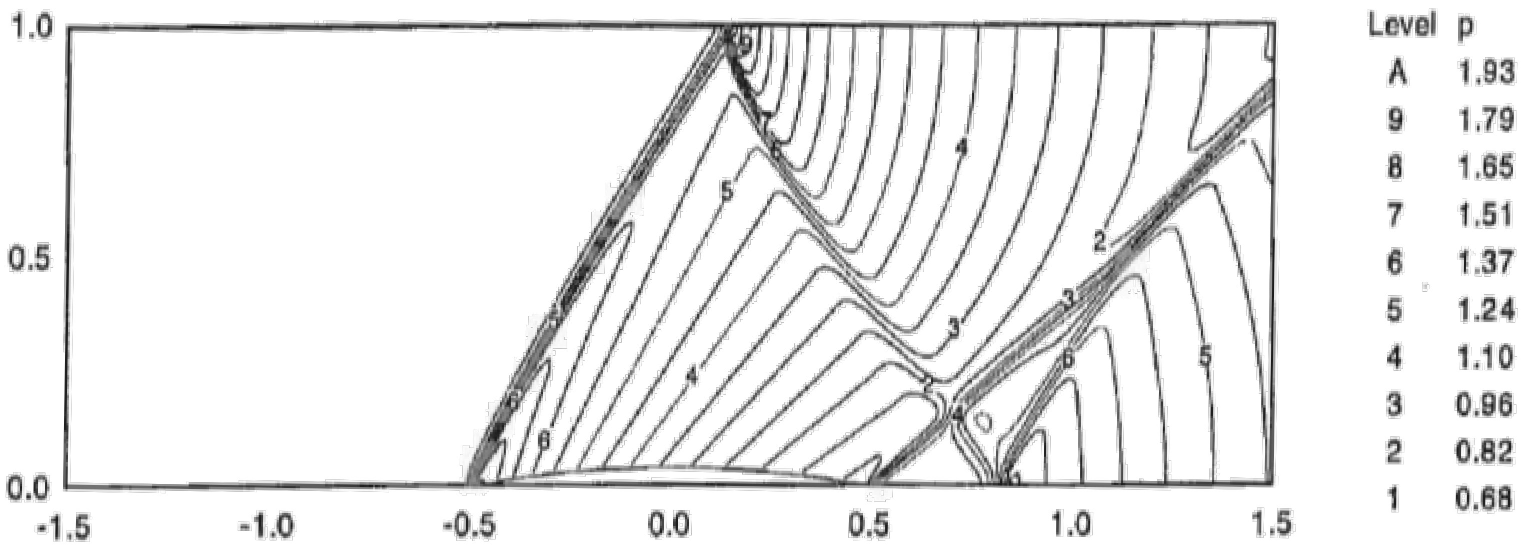


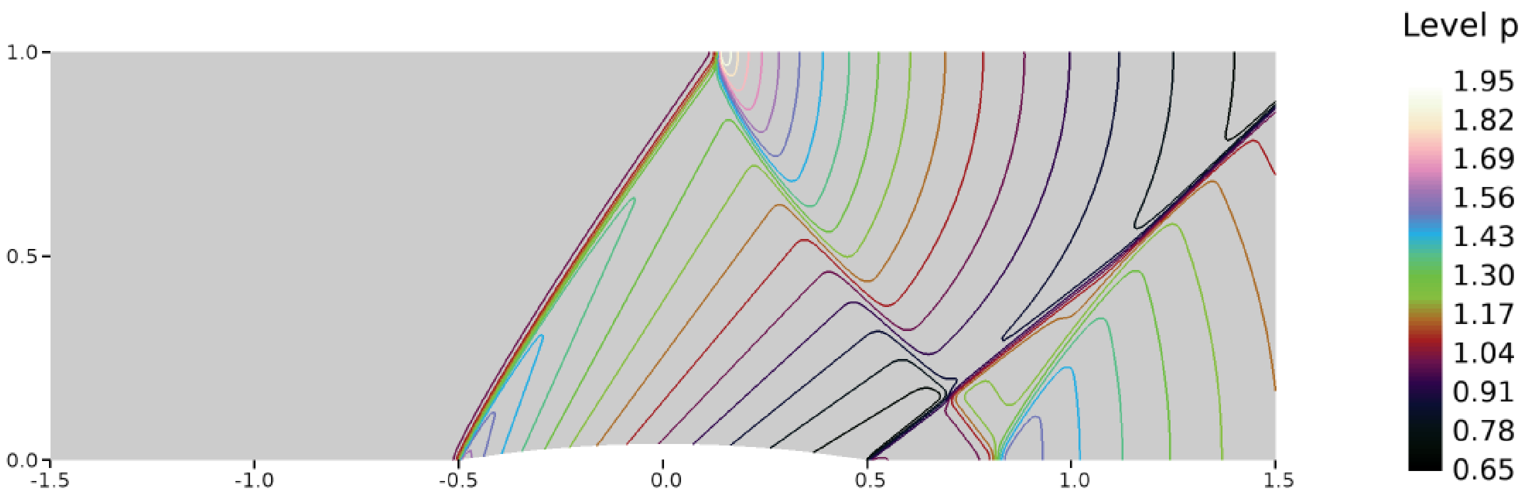
Supersonic inviscid flow over a 2D bump
Mach number and Prssure



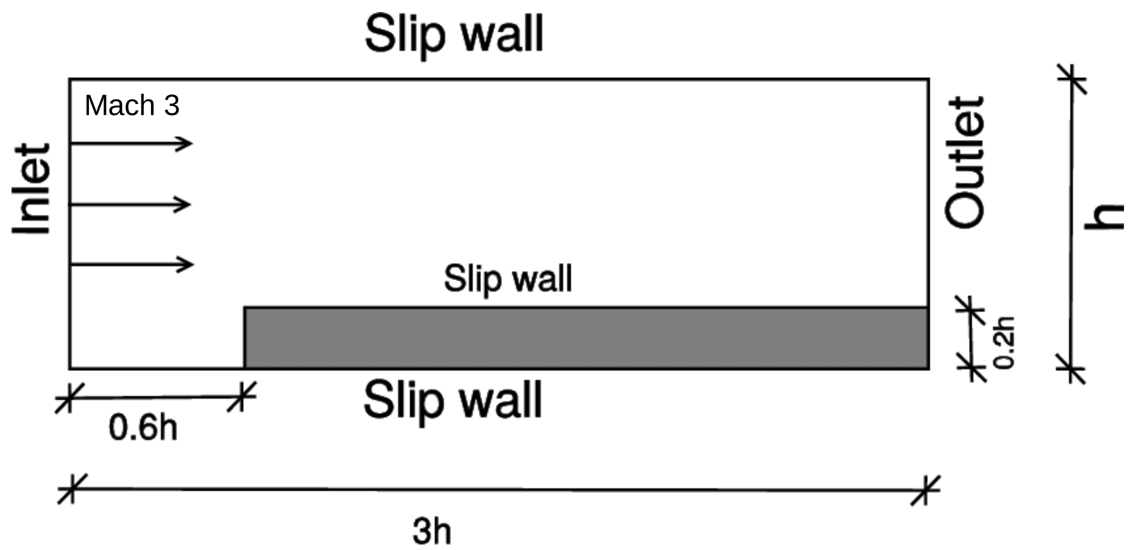


Supersonic inviscid flow over a 2D bump
Pressure



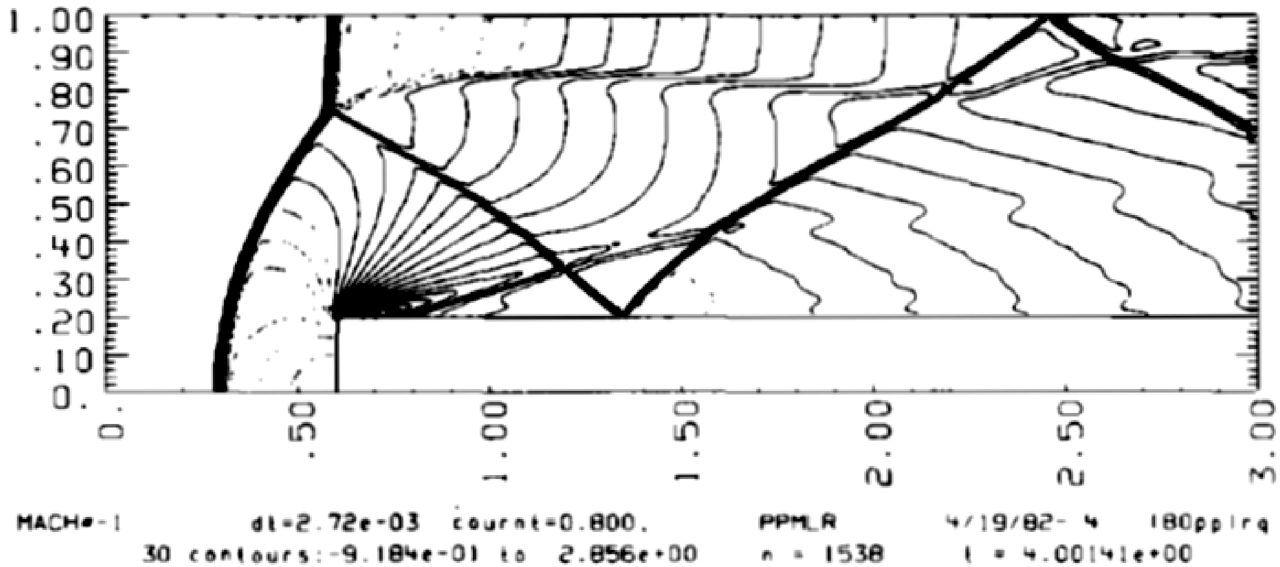


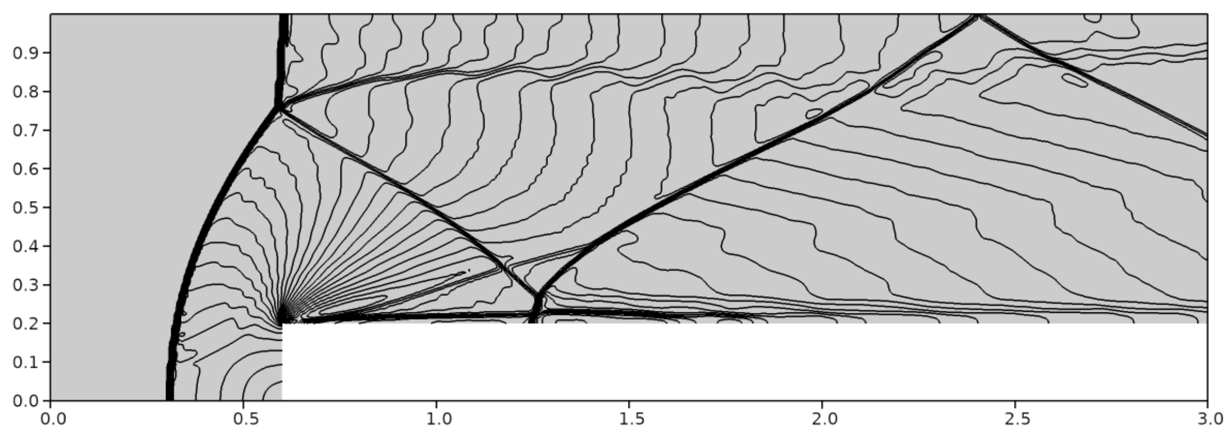
Transient Inviscid Supersonic Flow Past a Forward Facing Step



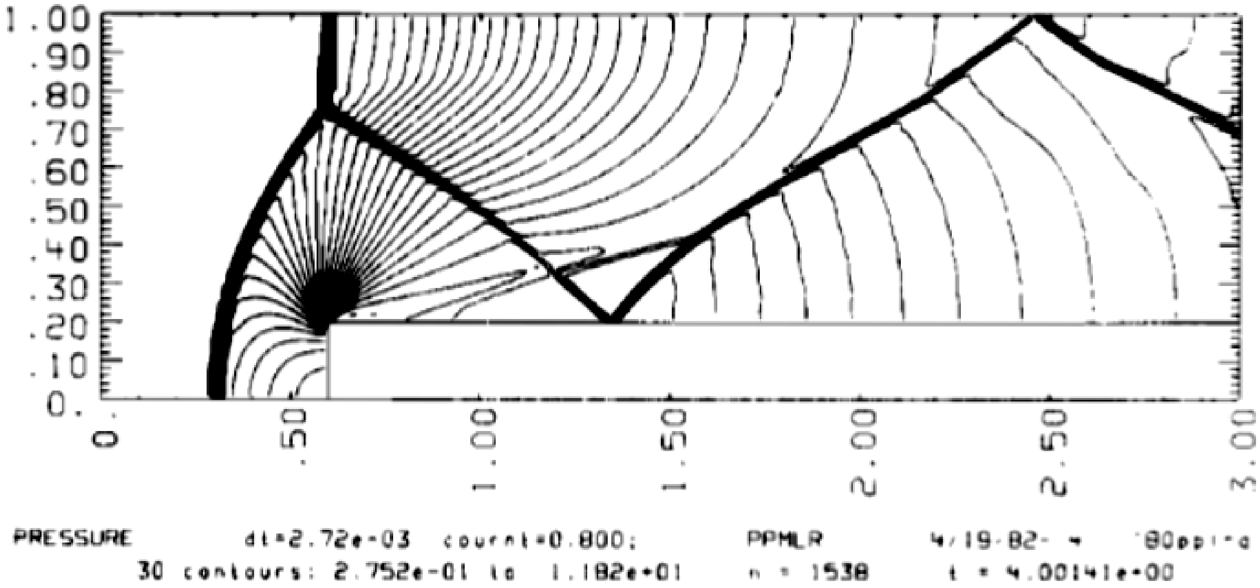
- Mach 3 case
- Comparison with solutions from Riemann solver
  - Woodward, P. and Colella, P. (1984) "The Numerical Simulation of Two-Dimensional Fluid Flow with Strong Shocks", Journal of Computational Physics.

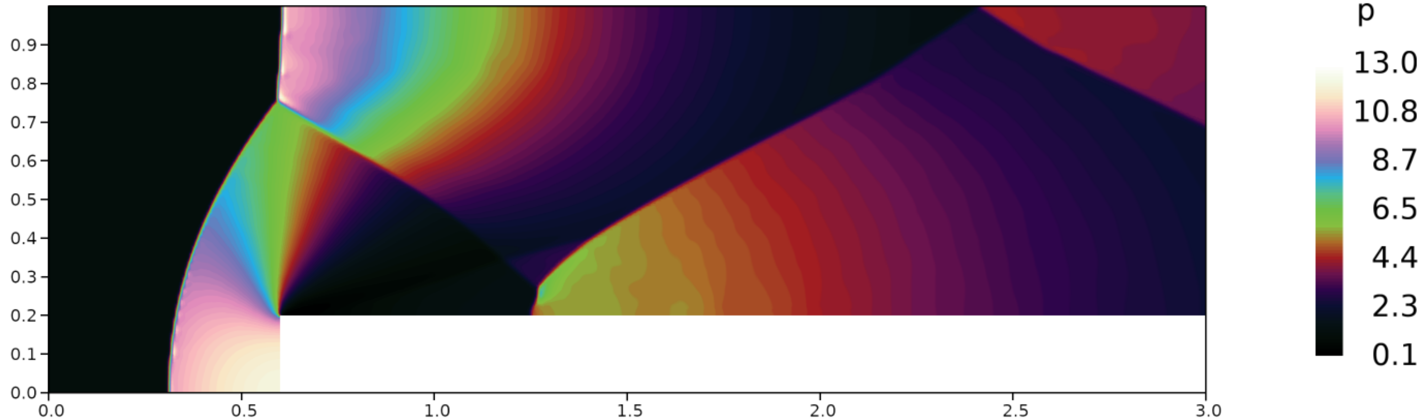
Forward Facing Step - Mach Number





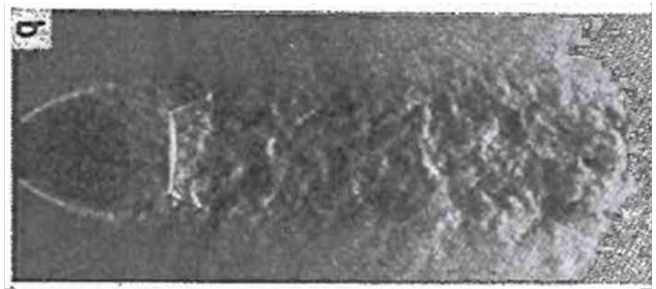
Forward Facing Step - Pressure



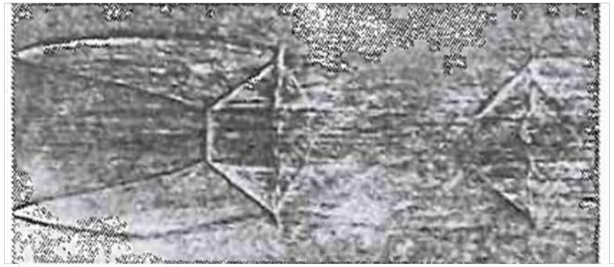


Ladenburg's supersonic under-expanded Jet

- Ladenburg's Axisymmetric Under-Expanded Supersonic Free-Jet
  - 10mm jet diameter at inlet; Mach 1 at jet exit; Experiment was done in the 40's.
    - Ladenburg, R., Van Voorhis, C.C. and Winckler, J (1949) "Intererometric Studies of Faster than Sound Phenomena, Part II Analysis of Supersonic Air Jets", Physical Review.
  - Jet density field was measured using shadowgraph interferometric technique.
  - Low Re K-epsilon turbulence model.

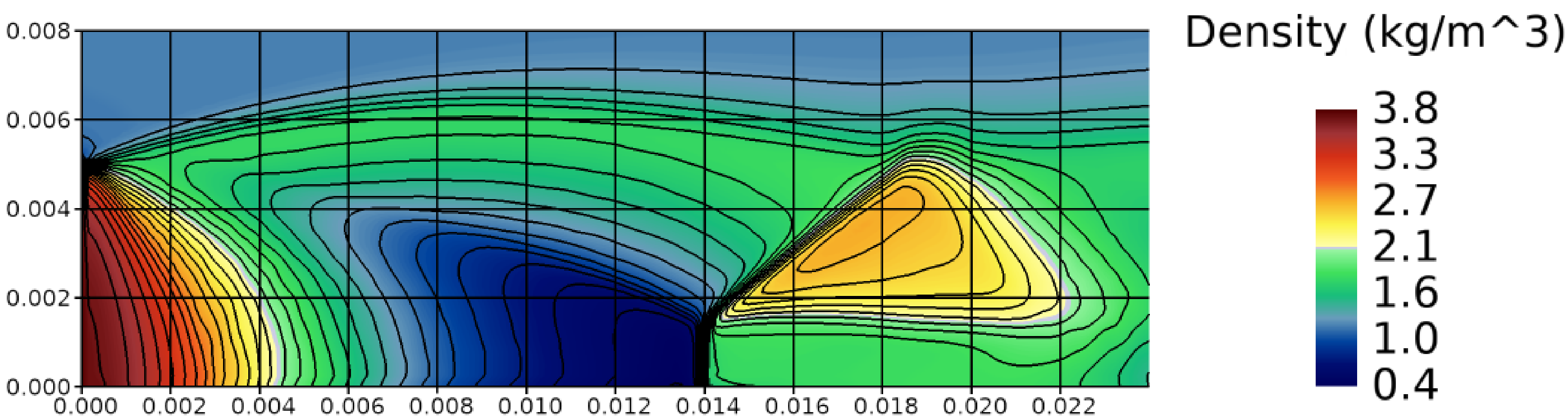


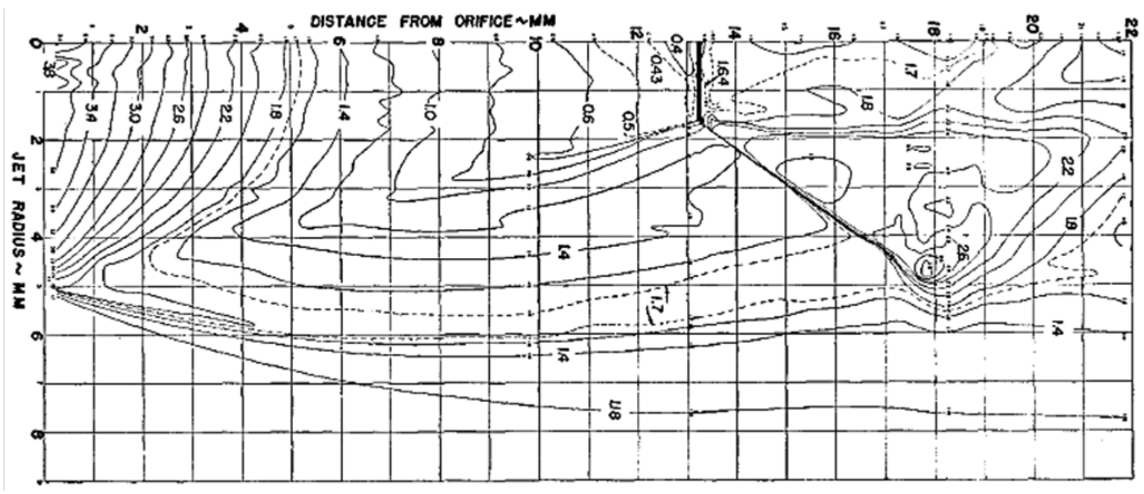
Schlieren Image of Ladenburg Jet 60 Psi Inlet Pressure



Shadowgram Image

Ladenburg’s jet - Density

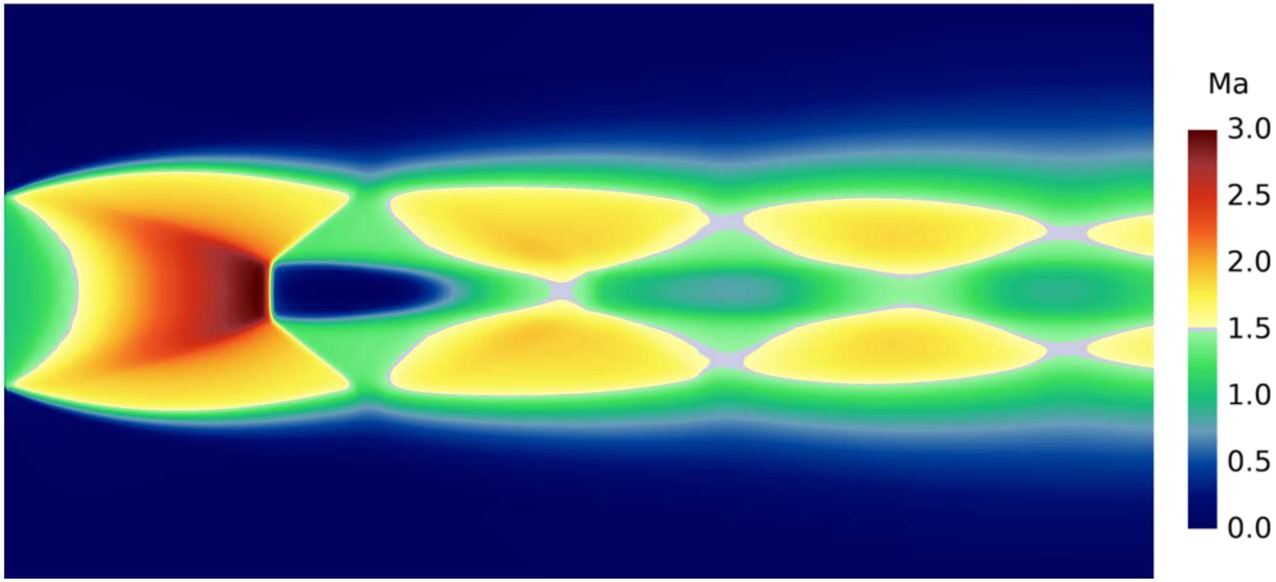


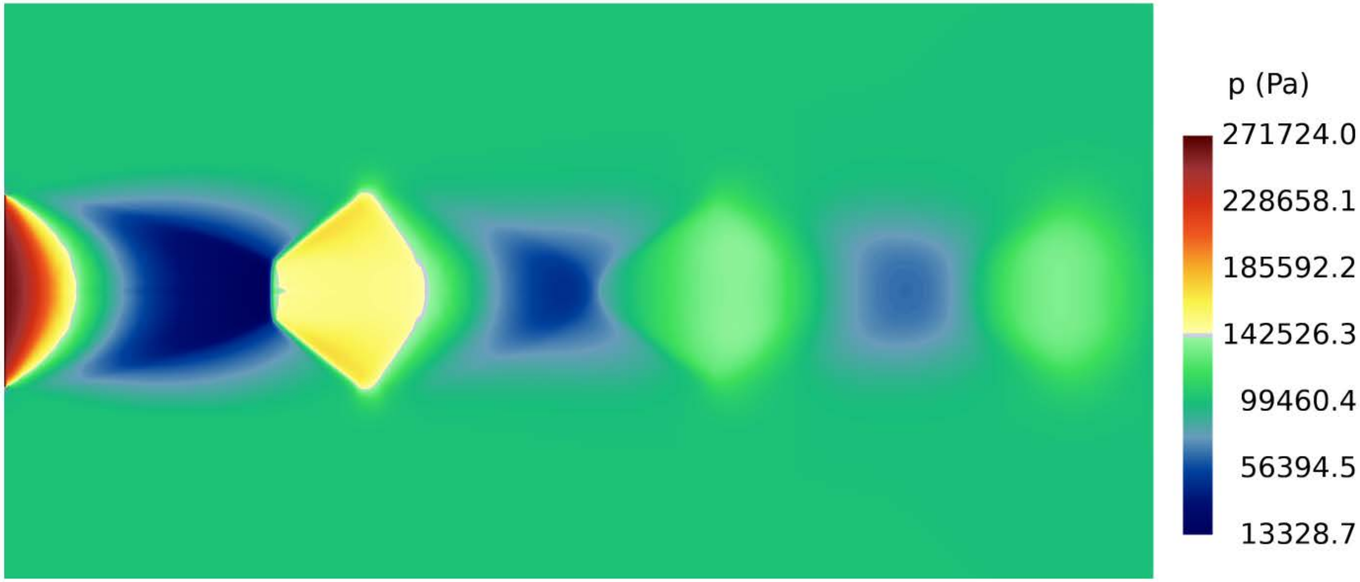


Density contour.

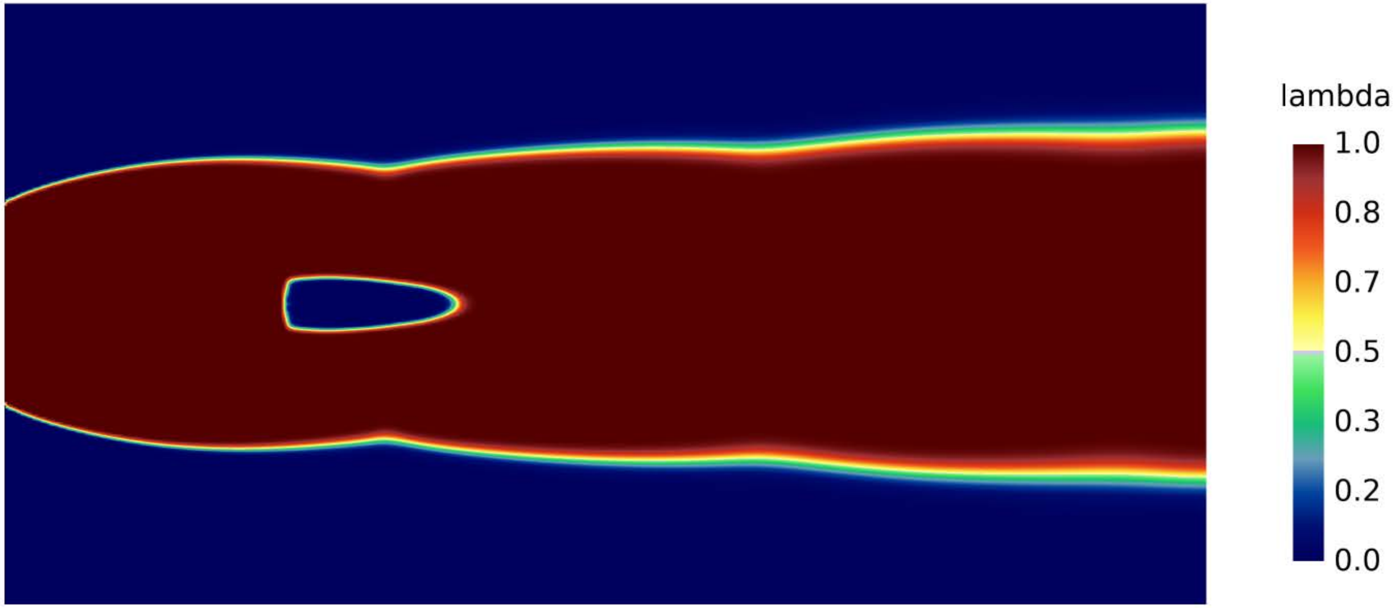
Interferometric Shadowgraph.

Ladenburg's jet – Mach number and Pressure





Ladenburg’s jet – Switching function



Conclusion

- A solver suitable for simulations of fluid flow for a wide range of Mach numbers developed.
- Uses KTNP scheme to keep shocks sharp.
  - Fixing deficiency of R-C interpolation at high speed.
- Verification and Validation tests have shown the solver is capable of computing flow for a wide range of Mach number.
- Flux formulation switching makes solver suitable for problems with region of high and low speed flows.

Questions

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