TIME SYMMETRY PRESERVING ADJOINT SOLVER IN EXTERNAL AERODYNAMICS

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Introduction

- Adjoint equations for the incompressible turbulent Navier-Stokes equations are presented
- The main characteristic of the proposed adjoint formulation is that it is time-symmetry preserving
- Consequence of time preservation is that the kinetic energy is conserved in this formulation
- Incompressible solver algorithm used for the solution of adjoint equations
- Surface sensitivity case study performed for the case of external aerodynamics of the Australian V8 supercar

Adjoint Approach to Design

- Governing equations for incompressible turbulent flows

$$\mathcal{A} = \begin{pmatrix} \partial_i u_i \\ \partial_t u_i + \Lambda u_i + \partial_i \hat{p} - f_i \end{pmatrix}$$

where Λ is the following operator

$$\Lambda = \partial_j u_j - \partial_j \left[(\nu + \nu_t) \left(\partial_j + \partial_i \right) \right]$$

- Density is absorbed in pressure. i.e. $\hat{p} = p/\rho$
- Turbulence models provide for the turbulent viscosity ν_t

Adjoint Approach to Design

- Adjoint equations are derived though definition of the Lagrangian $\mathcal{L}$

$$\mathcal{L} = \mathcal{J} + \langle \boldsymbol{Q}^*, \mathcal{A} \rangle$$

- Symbol $\langle\cdot,\cdot\rangle$ represent a scalar product of two functions
- $\boldsymbol{Q}^*$ is a vector of adjoint variables
- $\mathcal{J}$ is a measure of performance evaluated on the surface of the car
- With this definition a new Lagrangian is defined that uses the objective function and the system of governing equations
- Vector of adjoint variables plays a role of Lagrangian multipliers

Adjoint Approach to Design

- The Lagrangian $\mathcal{L}$ requires the linearization of the operator $\mathcal{A}$
- Linearization can be performed in many ways
- Here we choose to linearize the operator in such way that resulting adjoint system of equations preserves symmetry in time
- Navier-Stokes operator in quasi-linear form

$$\partial_t u_i + \left(\mathcal{N}(u_j)\right) u_i + \partial_i \hat{p} - f_j = 0$$

where the operator $\left(\mathcal{N}(u_j)\right) u_i$ is given by

$$\left(\mathcal{N}(u_j)\right) u_i = \Lambda u_i$$

Adjoint Approach to Design

- The resulting Lagrangian takes the following form

$$\mathcal{L} = \mathcal{J} + \int_{\Omega} (\hat{p}^{*}, u_i^{*}) \begin{pmatrix} \partial_i u_i \\ \partial_t u_i + \Lambda u_i + \partial_i \hat{p} - f_j \end{pmatrix} d\Omega$$

- Take the total variation of Lagrangian

$$\delta\mathcal{L}\left(\mathcal{P}, u_i \ , \hat{p}\right) = \delta_P \mathcal{L} + \delta_{u_i} \mathcal{L} + \delta_{\hat{p}} \mathcal{L}$$

Adjoint Approach to Design

- The resulting system of adjoint equations is obtained by performing the integration by parts

$$\mathcal{A}_1^* = \partial_i u_i^*,$$

$$\mathcal{A}_2^* = -\partial_t u_i^* - u_j \partial_j u_i^* + \cdots + \partial_j \left[(\nu + \nu_t) \left(\partial_j u_i^* + \partial_i u_j^* \right) \right] - \partial_i \hat{p}^* - f_i^*$$

- Adjoint system of equations is supplemented by the corresponding adjoint boundary conditions
- Boundary conditions are determined by the same procedure that was used to define the adjoint system of equations

Time-Symmetry Preserving Adjoint Equations

- Write Navier-Stokes equations in the operator form

$$\begin{pmatrix} 0 \\ \partial_t u_i \end{pmatrix} + \begin{pmatrix} \partial_i u_i \\ \Lambda u_i + \partial_i \hat{p} - f_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- Introduce new operators

$$\boldsymbol{B} = \begin{pmatrix} 0 \\ \partial_t u_i \end{pmatrix}$$

$$\boldsymbol{C} = \begin{pmatrix} \partial_i u_i \\ \Lambda u_i + \partial_i \hat{p} - f_i \end{pmatrix}$$

$$\boldsymbol{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Time-Symmetry Preserving Adjoint Equations

- Symbolic form of Navier-Stokes and adjoint equations take the following form

$$\boldsymbol{B}\partial_t \boldsymbol{Q} + \boldsymbol{C}\boldsymbol{Q} = \boldsymbol{0}$$

$$-\boldsymbol{B}\partial_t \boldsymbol{Q}^* + \boldsymbol{C}\boldsymbol{Q}^* = \boldsymbol{0}$$

- Form the inner product using $\boldsymbol{Q}$ and $\boldsymbol{Q}^*$

$$\langle \boldsymbol{B}\partial_t \boldsymbol{Q}, \boldsymbol{Q}^* \rangle + \langle \boldsymbol{C}\boldsymbol{Q}, \boldsymbol{Q}^* \rangle + \langle \boldsymbol{B}\partial_t \boldsymbol{Q}^*, \boldsymbol{Q} \rangle + \langle \boldsymbol{C}\boldsymbol{Q}^*, \boldsymbol{Q} \rangle = 0$$

Time-Symmetry Preserving Adjoint Equations

- Use the identity

$$\langle \boldsymbol{C}\boldsymbol{Q}, \boldsymbol{Q}^* \rangle = -\langle \boldsymbol{C}\boldsymbol{Q}^*, \boldsymbol{Q} \rangle$$

- The resulting system

$$\langle \boldsymbol{B}\partial_t \boldsymbol{Q}, \boldsymbol{Q}^* \rangle + \langle \boldsymbol{B}\partial_t \boldsymbol{Q}^*, \boldsymbol{Q} \rangle = 0$$

- The following quantity is constant in time

$$\frac{dE}{dt} = \frac{d}{dt}\langle \boldsymbol{B}\boldsymbol{Q}, \boldsymbol{Q}^* \rangle = \langle \boldsymbol{B}\partial_t \boldsymbol{Q}, \boldsymbol{Q}^* \rangle + \langle \boldsymbol{B}\partial_t \boldsymbol{Q}^*, \boldsymbol{Q} \rangle = 0$$

$$E = const$$

Numerical Algorithm

- Apply finite volume discretization to adjoint and Navier-Stokes equations

$$A_P^{u_i} u_{i,P}^{n+1} + \sum_l A_l^{u_i} u_{i,l}^{n+1} = f_i^{n+1} - \left(\delta_i \hat{p}^{n+1}\right)_P$$

$$u_{i,P}^{n+1} = \frac{1}{A_P^{u_i}} \left(f_i^{n+1} - \sum_l A_l^{u_i} u_{i,l}^{n+1} \right) - \frac{1}{A_P^{u_i}} (\delta_i \hat{p}^n)_P$$

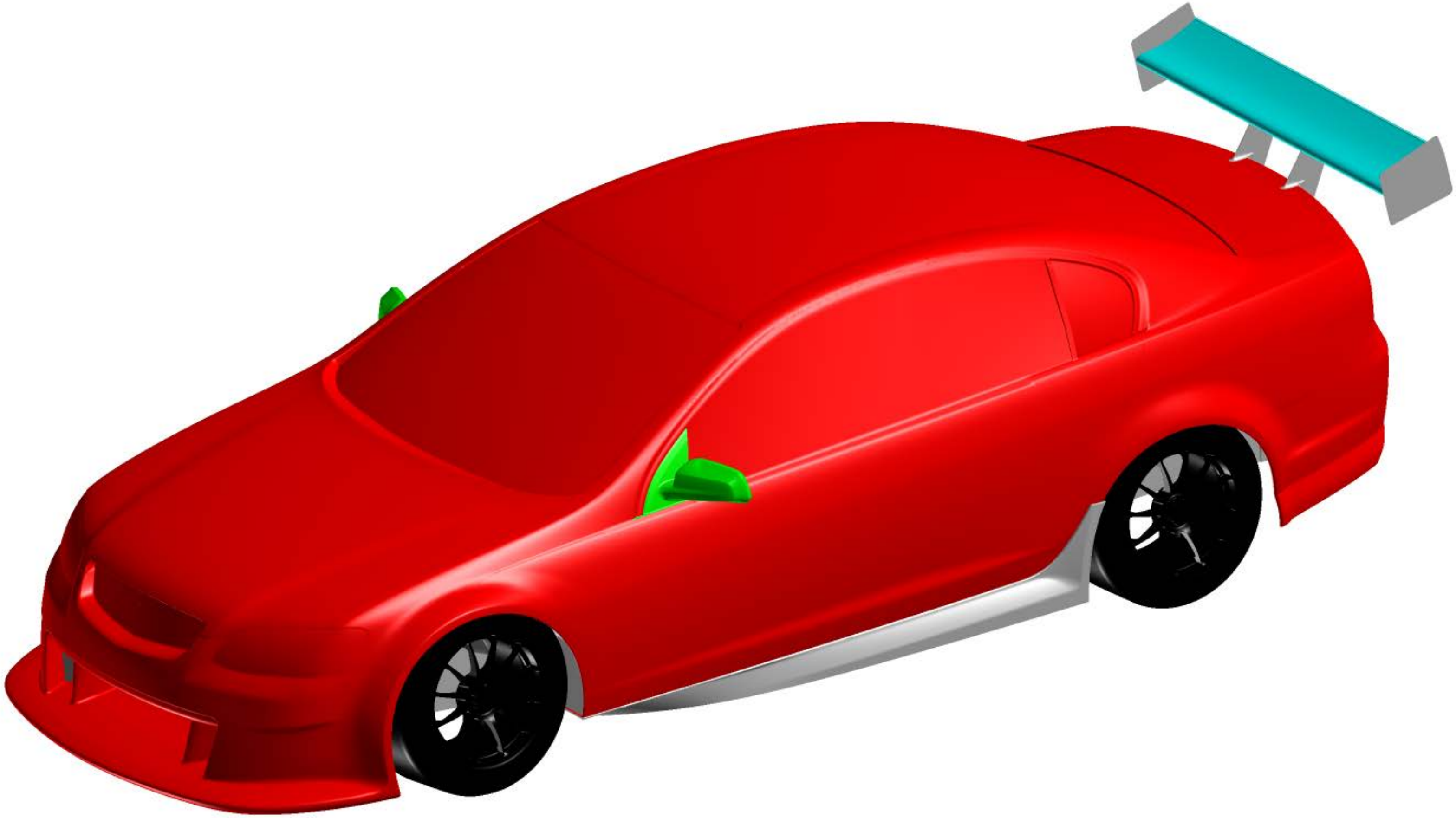
$$\delta_i \left[\frac{1}{A_P^{u_i}} \left(\delta_i \hat{p}^{n+1}\right)_P \right] = \delta_i \left(\tilde{u}_i^{\dagger}\right)_P$$

$$\tilde{u}_i^{\dagger} = f_i^{n+1} - \sum_l A_l^{u_i} u_{i,l}^{n+1}$$

$$\tilde{u}_{i,corr}^{\dagger} = \tilde{u}_i^{\dagger} - \left[\frac{1}{A_P^{u_i}} \left(\delta_i \hat{p}^{n+1}\right)_P \right]$$

V8 Supercar Case Study

- Primal flow field results



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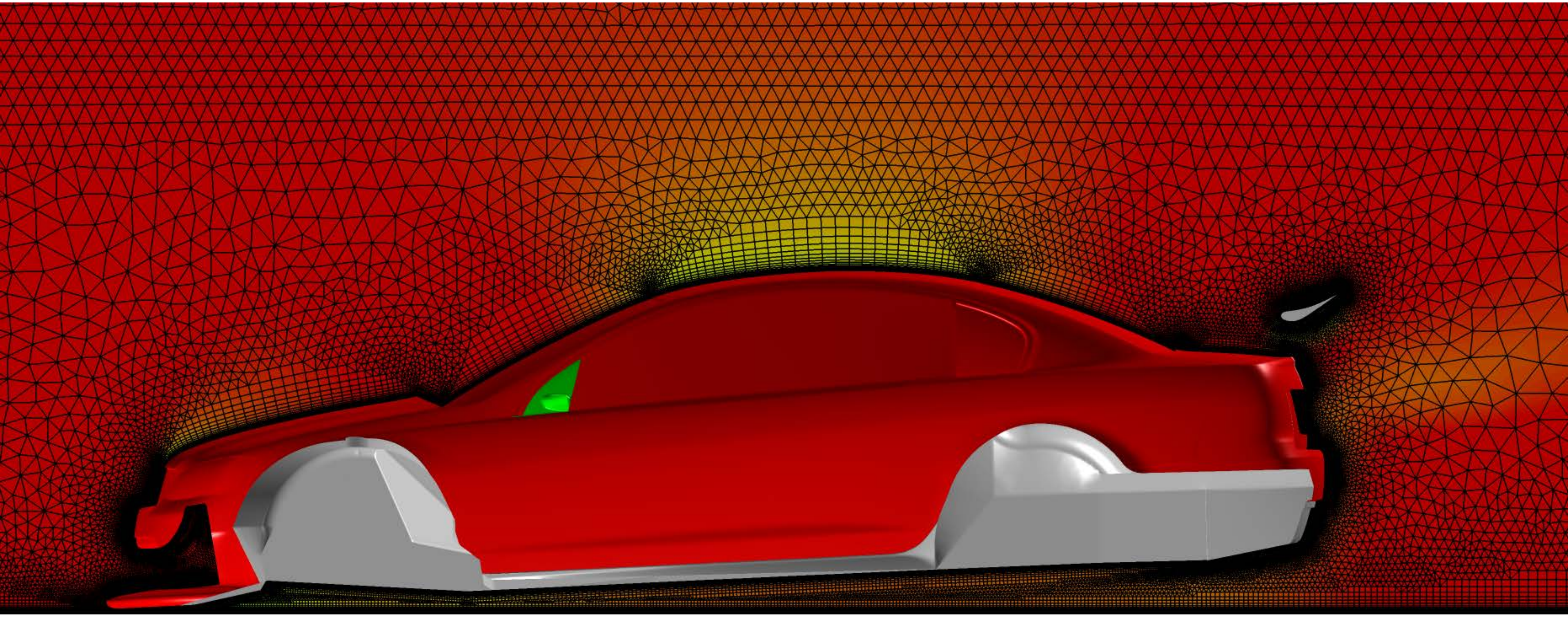

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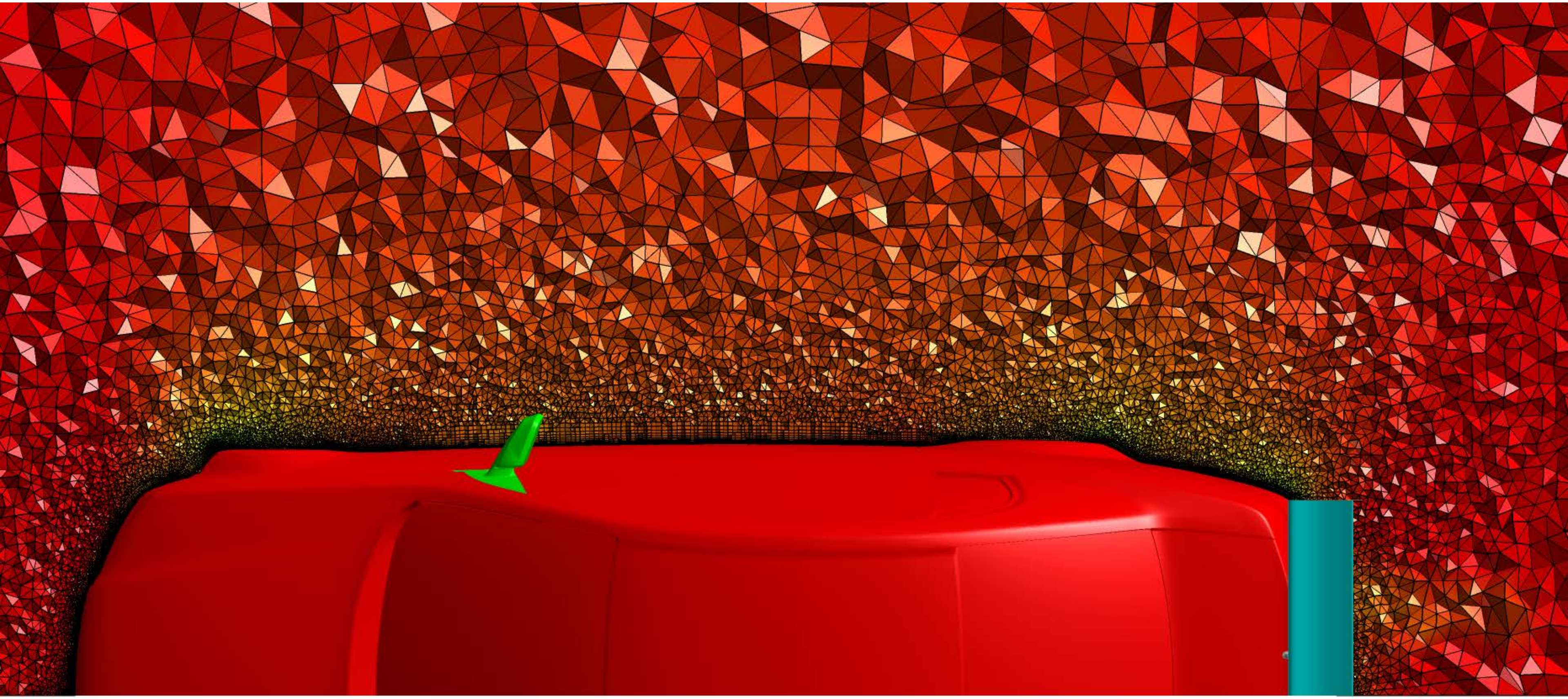

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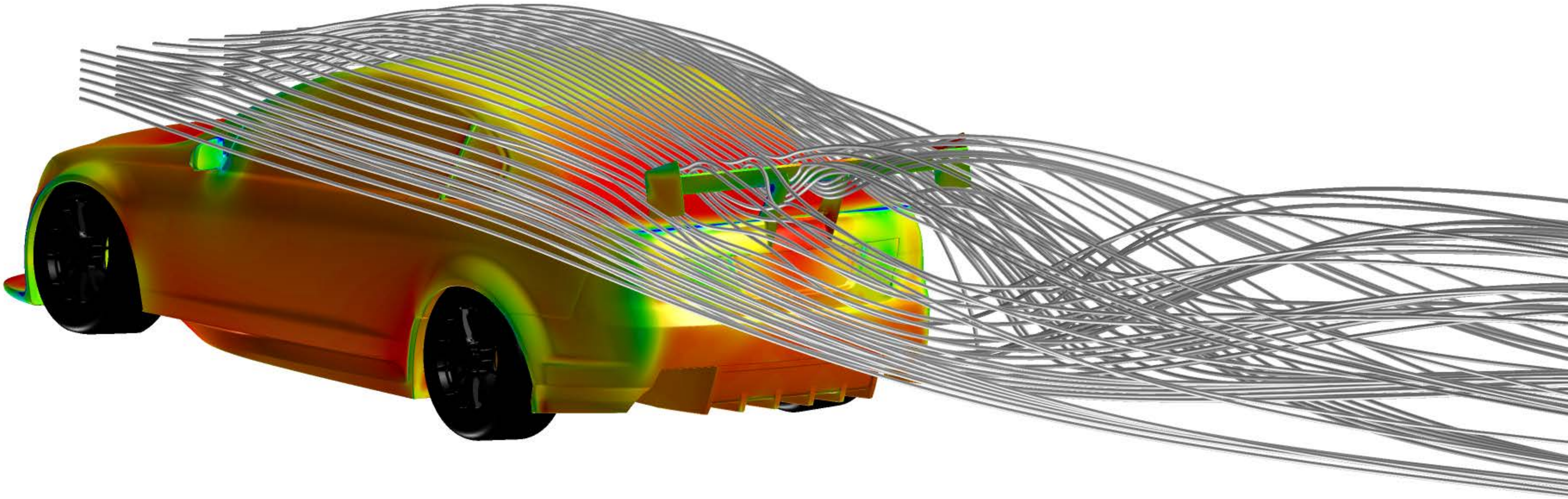

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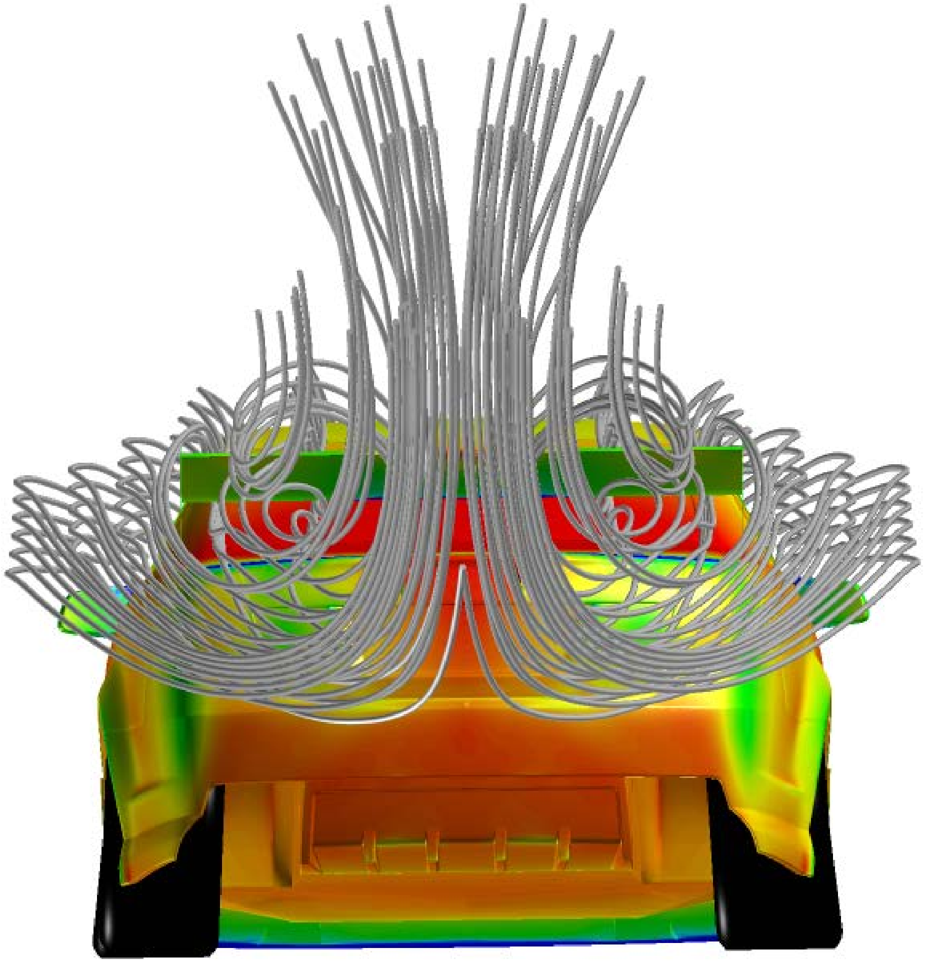

V8 Supercar Case Study

- Local force on the surface

$$F_i = -p\delta_{ij} + (\nu + \nu_t)\left(\partial_i u_j + \partial_j u_i\right) n_j dS$$

- Objective function

$$\mathcal{J} = \int_\Gamma F_i n_i dS$$

- Shape derivative is defined as

$$\delta_i \mathcal{J} = \int_\Gamma s_i n_i d\Gamma$$

where s_i is given by

$$s_i = \left[\left(u_j u_i^*\right) - \hat{p}^* \delta_{ij} + (\nu + \nu_t)\left(\partial_j u_i^* + \partial_i u_j^*\right)\right] n_j$$

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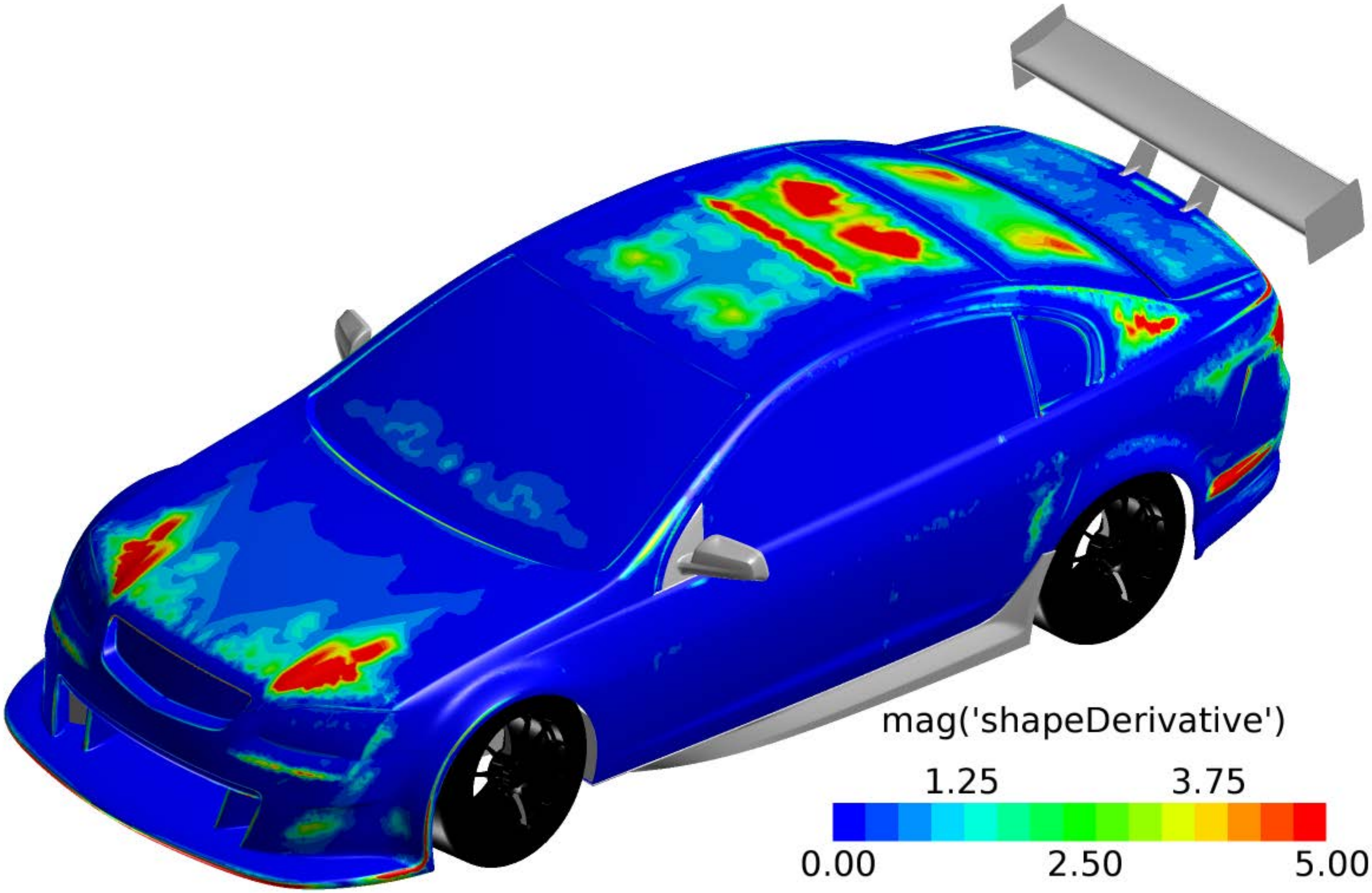

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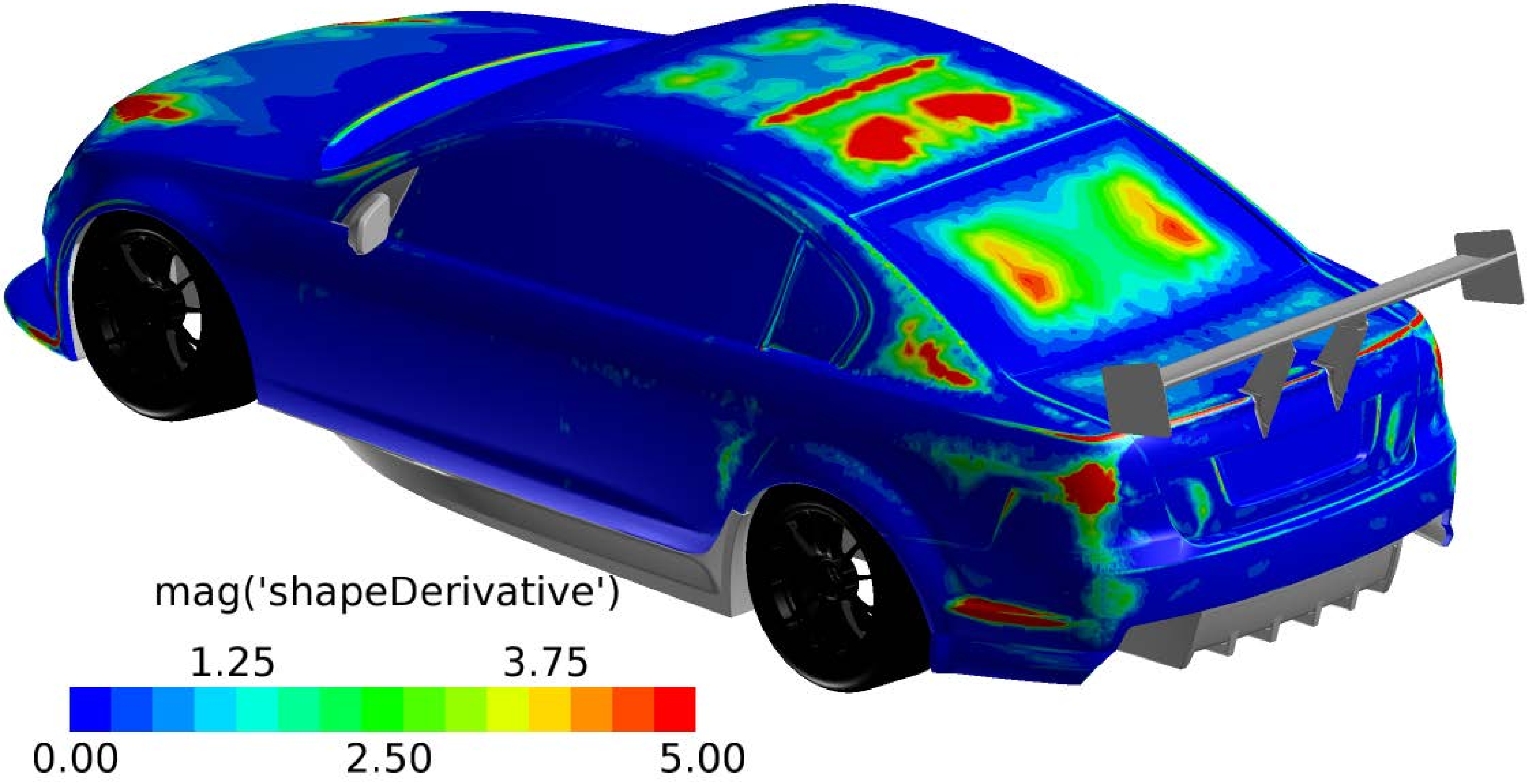

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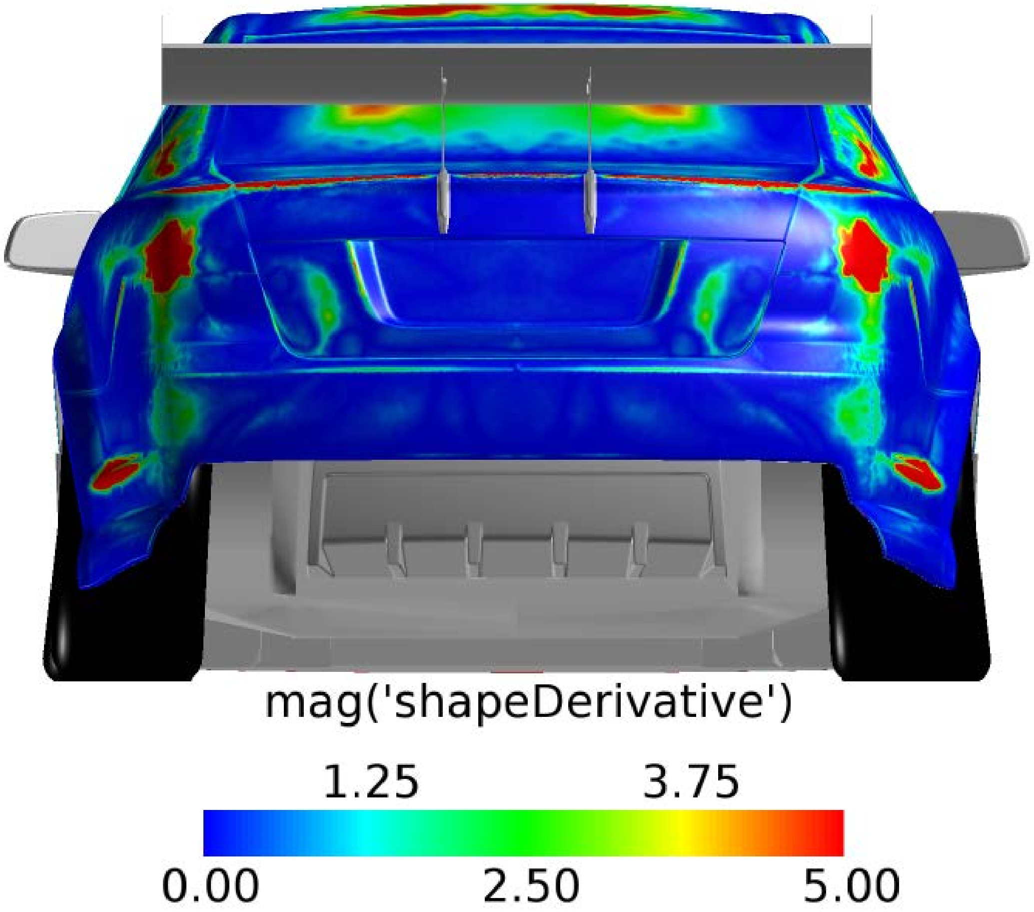

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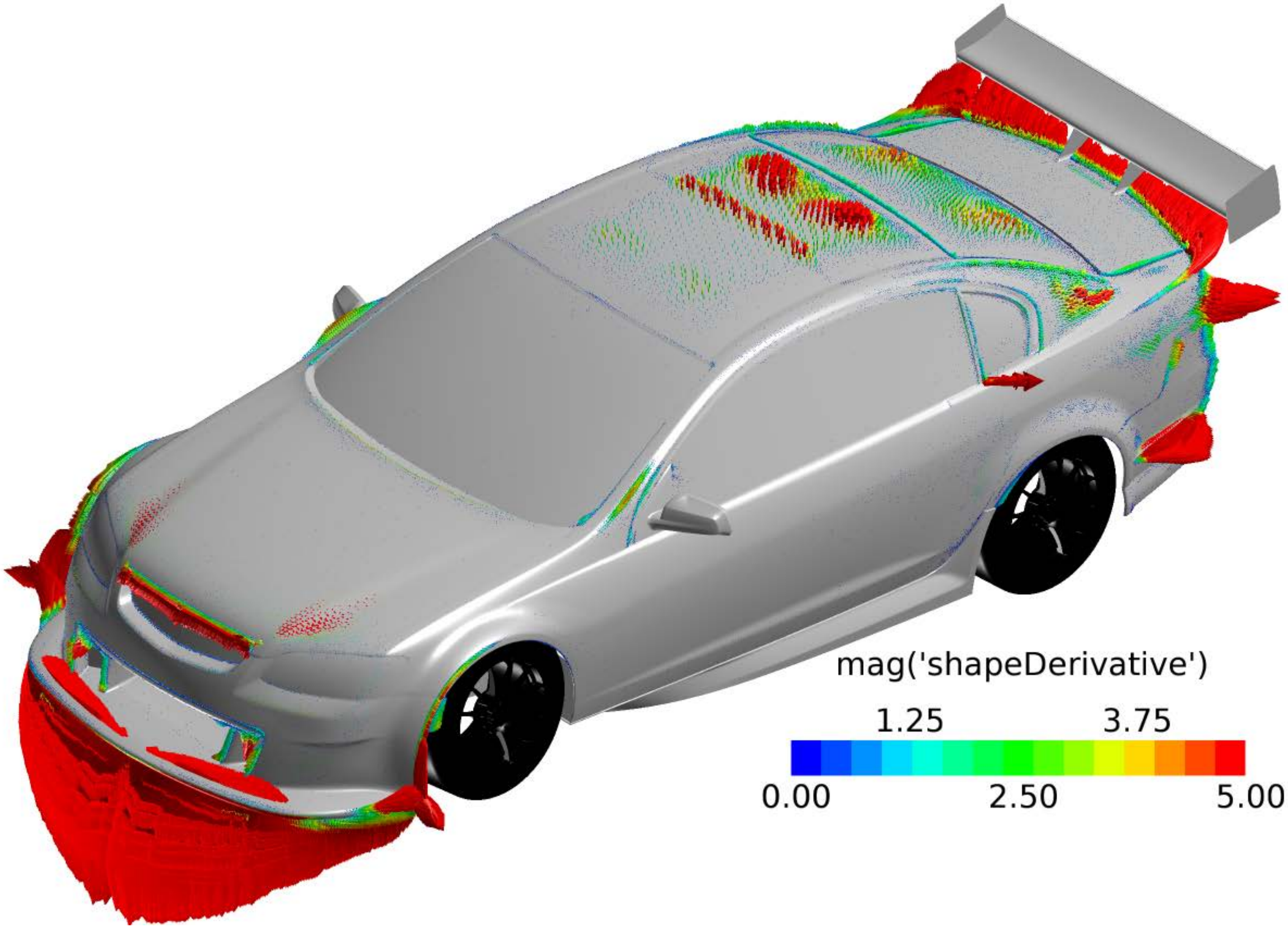

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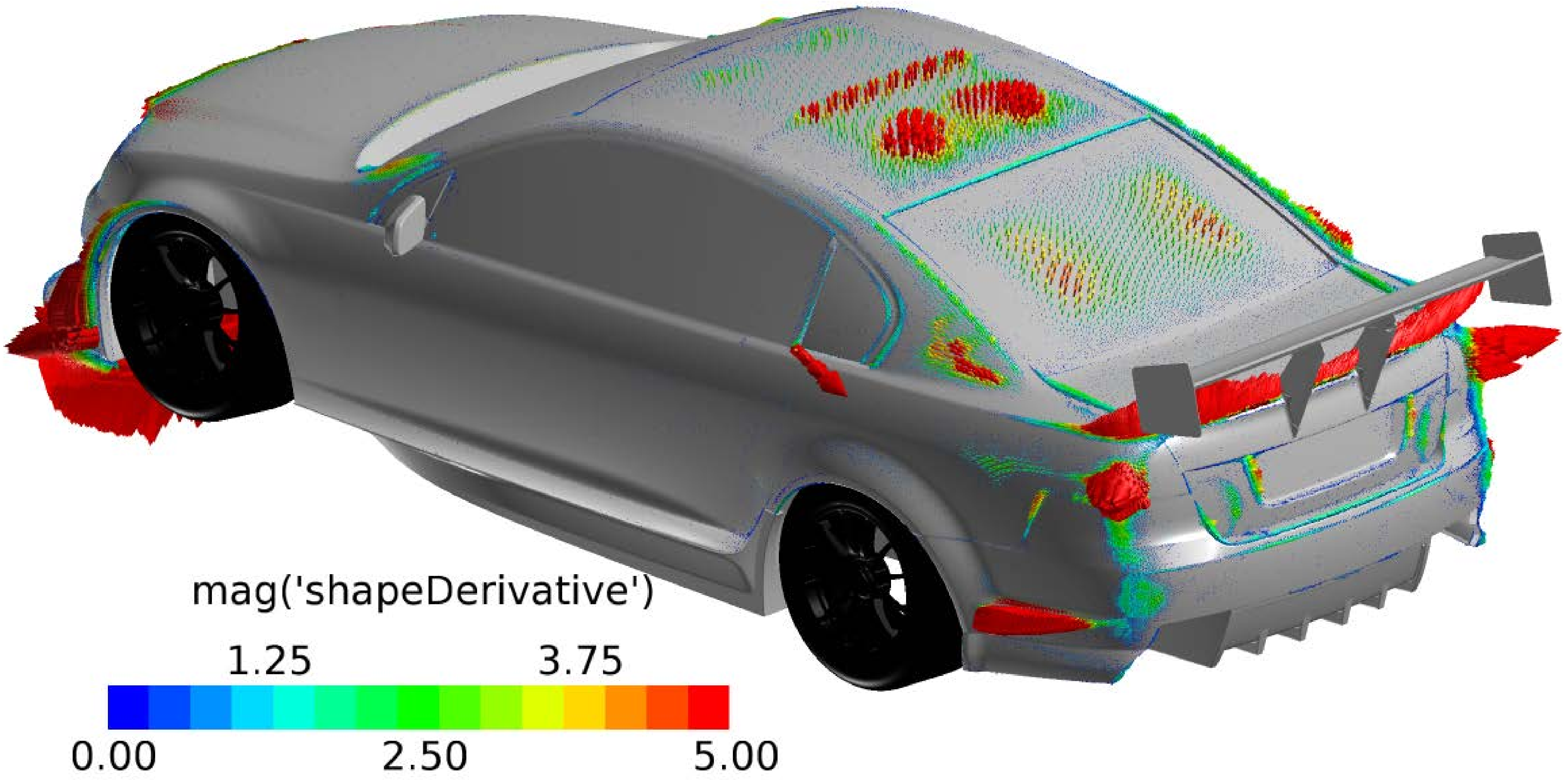

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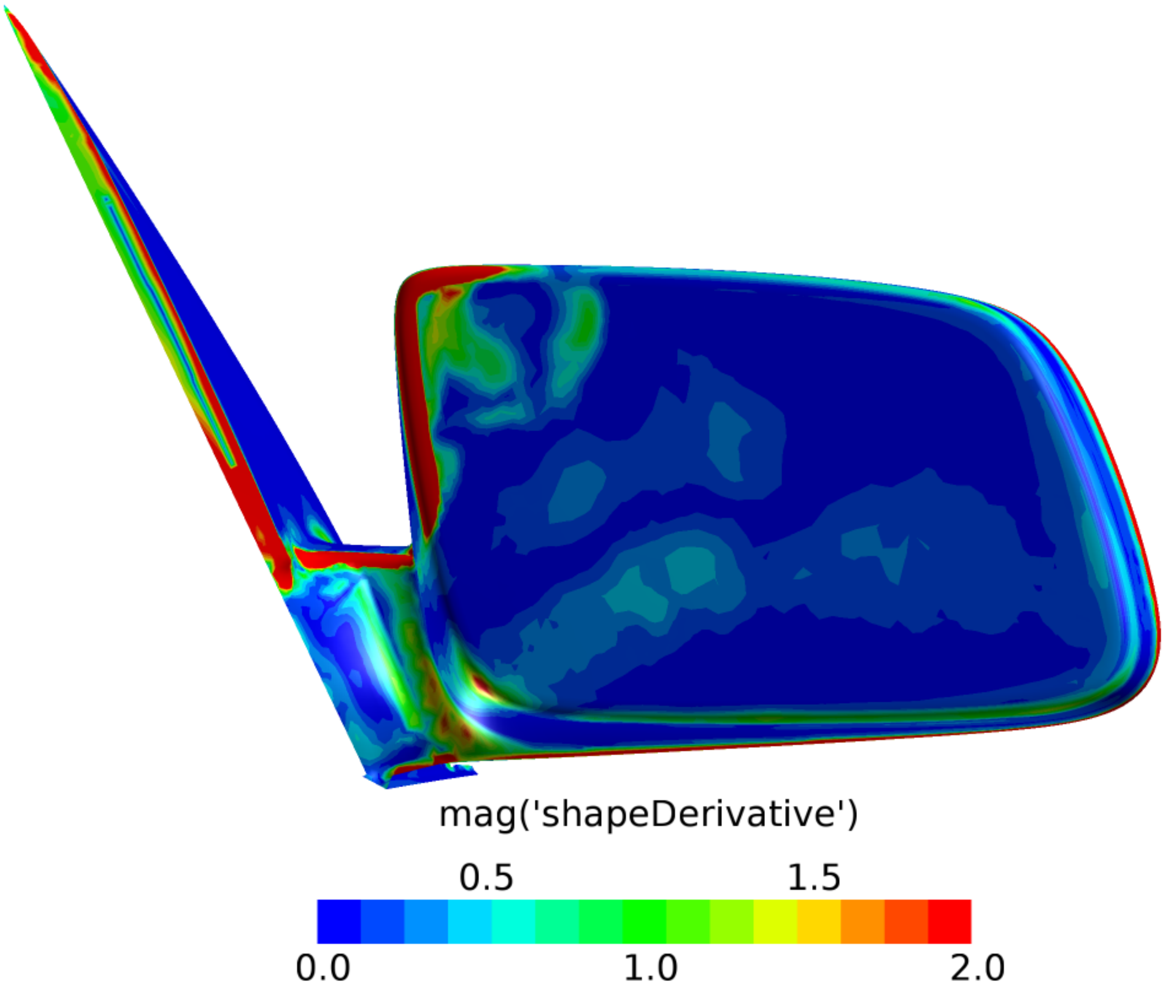

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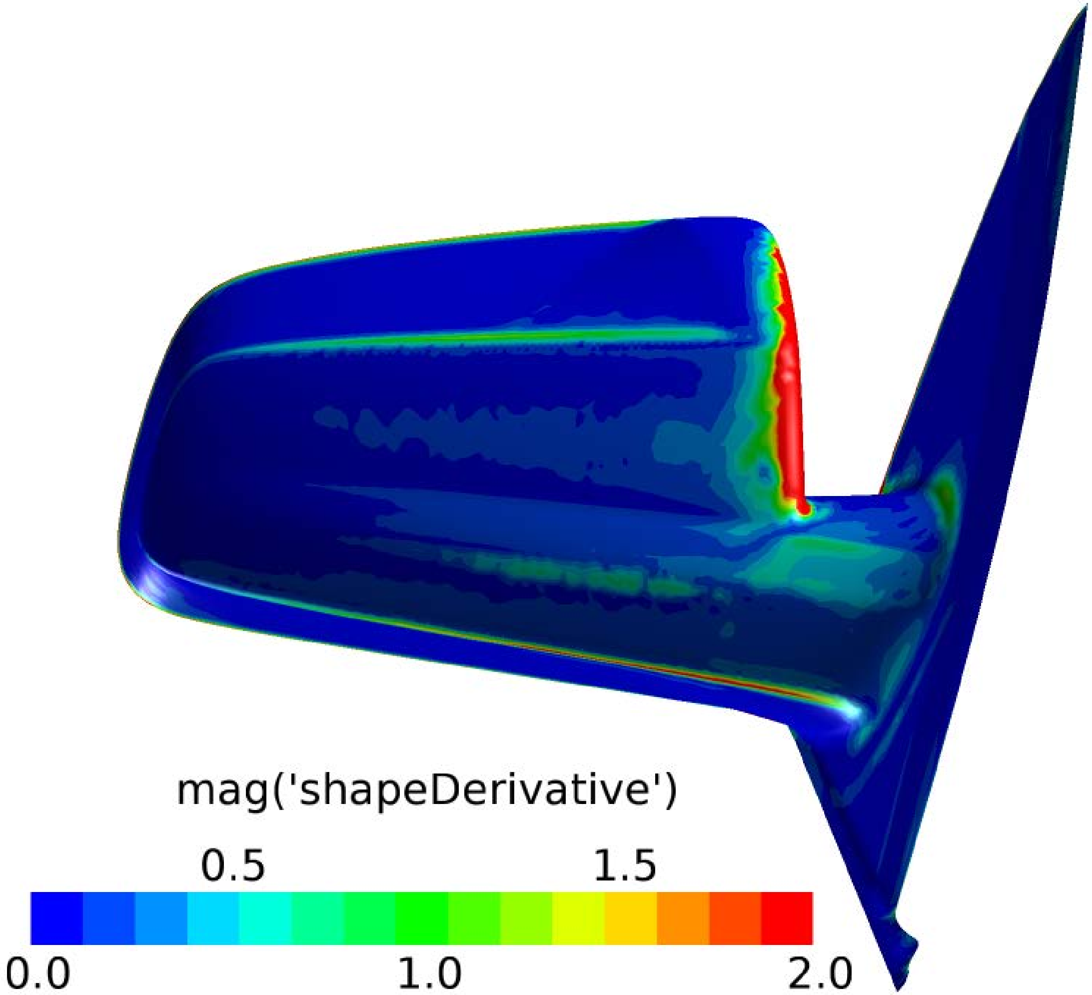

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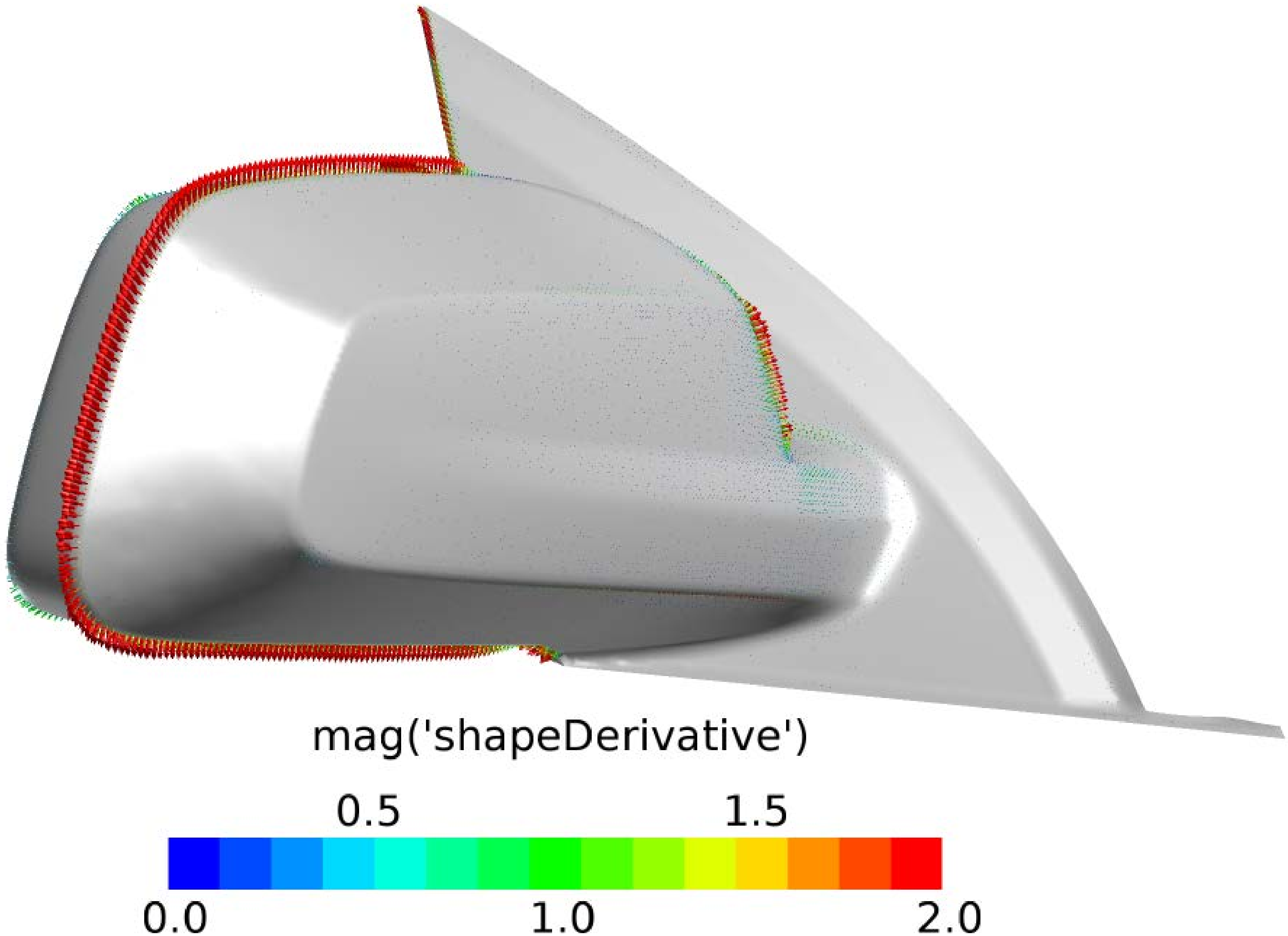

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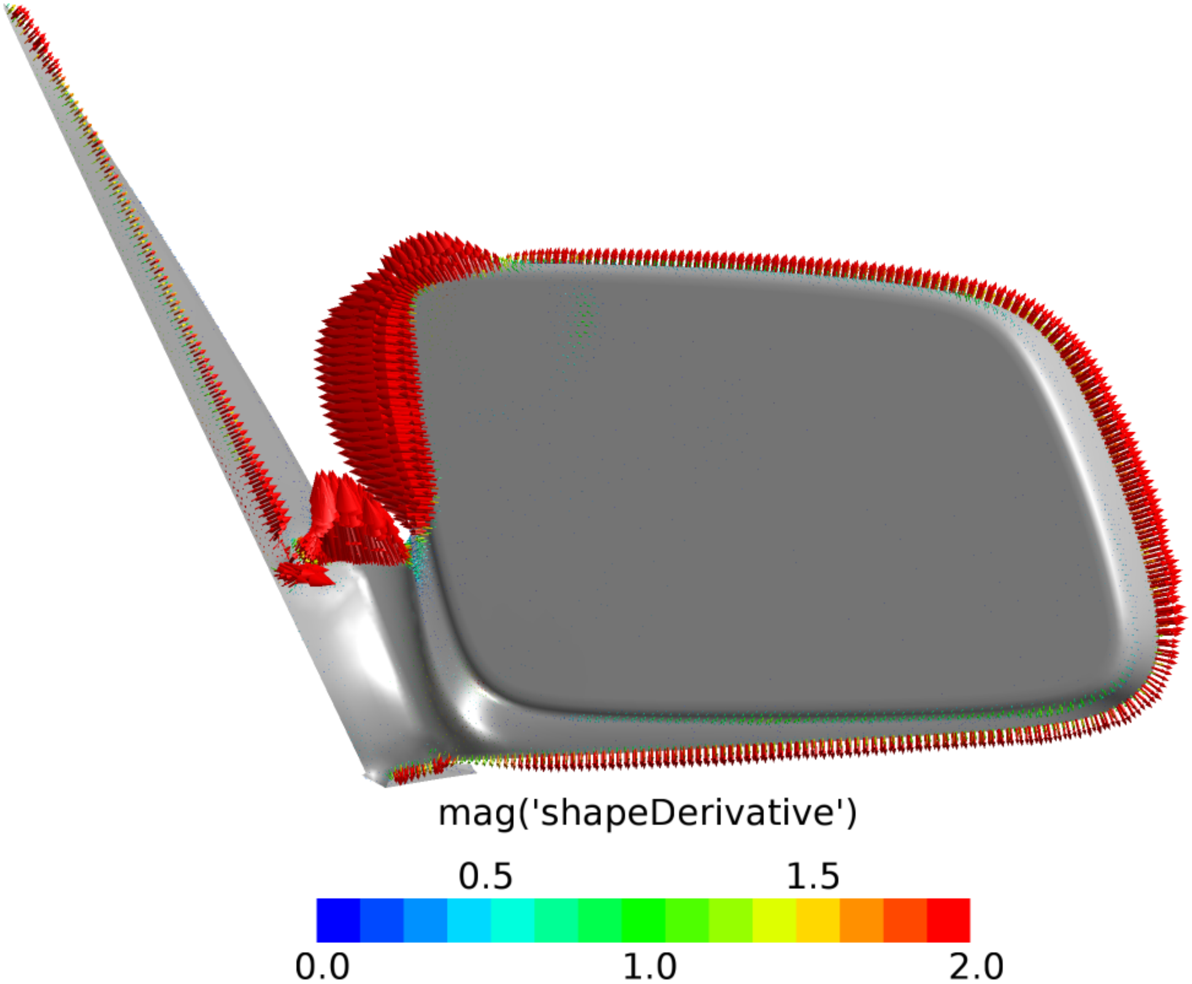

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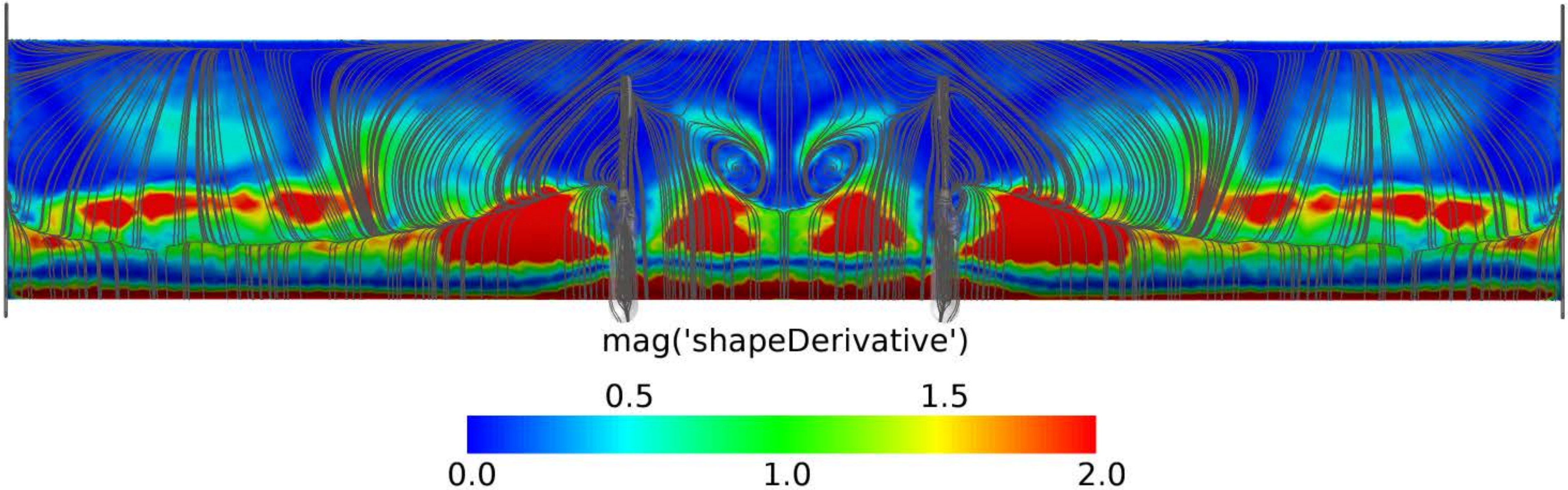

Conclusions

- Time-symmetry preserving adjoint system of equations formulation was presented
- The same underlying pressure-velocity coupling algorithm used for the flow solution was used to solve adjoint system of equations
- The resulting surface sensitivity vectors computed indicate the direction in which the shape of the car should be modified to get the maximum force change
- The local shape change does not necessarily coincide with the surface normal
- Shape morphing can be easily guided by the resulting sensitivity field to produce improved shape of the car